Reflectance & Lighting

Computer Vision I
CSE 252A
Lecture 6

Radiometry

- Read Chapter 4 of Ponce & Forsyth
- Homework 1 Assigned

Outline
- Solid Angle
- Irradiance
- Radiance
- BRDF
- Lambertian/Phong BRDF

Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
- The solid angle subtended by a patch area \( dA \) is given by
  \[
  \Omega = \frac{dA \cos \theta}{r^2}
  \]

Radiance

- Power is energy per unit time
- Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle
- Symbol: \( L(x, \theta, \phi) \)
- Units: watts per square meter per steradian: \( \text{w} / (\text{m}^2 \text{sr}) \)

Radiance transfer

What is the power received by a small area \( dA_2 \) at distance \( r \) from a small area \( dA_1 \) emitting radiance \( L \)?

From definition of radiance

\[
L = \frac{P}{(dA \cos \theta)d\omega}
\]

From definition of solid angle

\[
d\omega = \frac{dA \cos \theta}{r^2}
\]

\[
P = LdA_1 \cos \theta_1 d\omega_1 \rightarrow 2
\]

\[
= L \frac{dA_1 dA_2 \cos \theta_1 \cos \theta_2}{r^2}
\]

Power emitted from patch, but radiance in direction different from surface normal.
Irradiance

- How much light is arriving at a surface?
- Sensible unit is irradiance.
- This is a function of incoming angle.

A surface experiencing radiance \( L(x, \theta, \phi) \) coming in from solid angle \( d\omega \) experiences irradiance:

\[
E(x) = L(x, \theta, \phi) \cos \theta d\omega
\]

- Crucial property: Total power arriving at the surface is given by adding irradiance over all incoming angles.

\[
\text{Total power} = \int \int \int E(x, y, z, \lambda) \, dx \, dy \, dz \, d\lambda \, d\omega
\]

Intermezzo: Camera’s sensor

- Measured pixel intensity is a function of irradiance integrated over
  - pixel’s area
  - over a range of wavelengths

For some time

\[
I = \int \int \int E(x, y, z, \lambda) \, dx \, dy \, dz \, d\lambda
\]

Light at surfaces

Many effects when light strikes a surface could be:

- transmitted
  - Skin, glass
- reflected
  - mirror
- scattered
  - milk
- travel along the surface and leave at some other point
- absorbed
  - sweaty skin

Assume that

- surfaces don’t fluoresce
  - e.g. scorpions, detergents
- surfaces don’t emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point

BRDF

With assumptions in previous slide

- Bi-directional Reflectance Distribution Function
  \( \rho(\theta, \phi, \theta', \phi') \)

- Ratio of incident irradiance to emitted radiance

Function of:
- Incoming light direction: \( \theta, \phi \)
- Outgoing light direction: \( \theta', \phi' \)

\[
\rho(\theta, \phi, \theta', \phi') = \frac{L(\theta, \phi, \theta', \phi')}{E(\theta, \phi) \cos \theta'} d\omega
\]
Properties of BRDFs

1. Non-negative: \( \rho(\theta_\text{in}, \phi_\text{in}; \theta_\text{out}, \phi_\text{out}) \geq 0 \)

2. Helmholtz Reciprocity Principle:
   \[ \rho(\theta_\text{in}, \phi_\text{in}; \theta_\text{out}, \phi_\text{out}) = \rho(\theta_\text{out}, \phi_\text{out}; \theta_\text{in}, \phi_\text{in}) \]

3. Total energy leaving a surface must be less than total energy arriving at the surface

\[ \int \int \int_{\Omega} \rho(\theta, \phi; \theta_\text{out}, \phi_\text{out}) \cos \theta \Theta \Phi \ d\Theta \ d\Phi \ d\Theta \geq \int \int \int_{\Omega} \rho(\theta, \phi; \theta_\text{in}, \phi_\text{in}) \cos \theta \Theta \Phi \ d\Theta \ d\Phi \ d\Theta \]

Surface Reflectance Models

Common Models
- Lambertian
- Phong
- Physics-based
  - Specular
    - Blinn 1977
    - Cook-Torrance 1982
    - Ward 1992
  - Diffuse
    - Hanrahan, Kurgan 1993
  - Generalized Lambertian
    - Blinn 1992
  - Thoroughly Pitted Surfaces
    - Ward 1992
- Phenomenological
  - Koenderink, Van Doorn 1996

Arbitrary Reflectance
- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement
  - Marschner et al. 1999
  - Dana et al. 1999

Isotropic BRDF

\[ f(\theta_\text{O}, \phi_\text{O}, \theta_\text{i}, \phi_\text{i}) = f(\theta_\text{O}, \theta_\text{i}, \phi_\text{i} - \phi_\text{O}) \]

From Hertzmann & Seitz, CVPR’03

Anisotropic BRDF

From Hertzmann & Seitz, CVPR’03

Lambertian (Diffuse) Surface

- BRDF is a constant called the albedo
- Emitted radiance is NOT a function of outgoing direction -- i.e., constant in all directions.
- For lighting coming in single direction, emitted radiance is proportional to cosine of the angle between normal and light direction

\[ L_\text{i} = N \cdot \omega_\text{i} \]
Specular Reflection: Smooth Surface

Rough Specular Surface

Phong Model

Gonioreflectometers

Gonioreflectometers

Gonioreflectometers

• Three degrees of freedom spread among light source, detector, and/or sample

• Can add fourth degree of freedom to measure anisotropic BRDFs
Ward’s BRDF Measurement Setup

- Collect reflected light with hemispherical (should be ellipsoidal) mirror [SIGGRAPH 92]

Marschner’s Image-Based BRDF Measurement

- For uniform BRDF, capture 2-D slice corresponding to variations in normals

Light sources and shading

- How bright (or what colour) are objects?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
- Similar to irradiance

Standard nearby point source model

\[ p_2(x) \frac{N(x) \cdot S(x)}{r(x)^2} \]

- \( N \) is the surface normal
- \( p \) is diffuse (Lambertian) albedo
- \( S \) is source vector - a vector from \( x \) to the source, whose length is the intensity term
  - works because a dot-product is basically a cosine
Line sources

Radiosity due to line source varies with inverse distance, if the source is long enough.

Area sources

- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source.
- Change variables and add up over the source.
- See Forsyth & Ponce or a graphics text for details.

Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer.
  - The sun doesn’t vary for any reasonable meaning of closer.
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn’t vary much, and the distance doesn’t vary much either, and we can roll the constants together to get:

$$\rho(x)(N(x) \cdot S(x))$$

Lighting at infinity

- Direction is a three vector $s$, with $|s| = 1$.
- Described as function on a sphere: radiance as a function of direction $r(s)$.
- Single point source is a delta function at some direction.
- Multiple point sources: sum of delta functions.

Diffuse lighting at infinity: Spherical Harmonics

Green: Positive
Blue: Negative

Conversely, the light emitted at a given point also is a function on a 2-D space (sphere).
Conversely, the set of light rays emitted from all points...

Radiance properties

- In free space, radiance is constant as it propagates along a ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.

\[
\frac{d\Phi_1}{d\omega_1} = \frac{d\Phi_2}{d\omega_2} = \frac{dA_1}{dA_2} = \frac{d\omega_1}{d\omega_2}
\]

Light Field/Lumigraph Main Idea

- In free space, the 5-D plenoptic function can be reduced to a 4-D function (radiance) on the space of light rays.
- Camera images measure the radiance over a 2-D set – a 2-D subset of the 4-D light field.
- Rendered images are also a 2-D subset of the 4-D lumigraph.
- Likewise, the space of emitted lighting can be 4-D.
- Special lights like point source, line sources, area sources are special cases.

Shadows cast by a point source

- A point that can’t see the source is in shadow
- For point sources, the geometry is simple

Area Source Shadows

1. Fully illuminated
2. Penumbra
3. Umbra (shadow)
Shading models

Local shading model
- Surface has incident radiance due only to sources visible at each point
- Advantages:
  - Often easy to manipulate, expressions easy
  - Supports quite simple theories of how shape information can be extracted from shading
- Used in vision & real-time graphics

Global shading model
- Surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
- Advantages:
  - Usually very accurate
- Disadvantage:
  - Extremely difficult to infer anything from shading values
- Rarely used in vision, often in photorealistic graphics

What’s going on here?

- Local shading model is a poor description of physical processes that give rise to images
  - Because surfaces reflect light onto one another
  - This is a major nuisance; the distribution of light (in principle) depends on the configuration of every radiator; big distant ones are as important as small nearby ones (solid angle)
- The effects are easy to model
- It appears to be hard to extract information from these models

At the top, geometry of a gutter with triangular cross-section; below, predicted radiosity solutions, scaled to lie on top of each other, for different albedos of the geometry. When albedo is close to zero, shading follows a local model; when it is close to one, there are substantial reflexes.

Irradiance observed in an image of this geometry for a real white gutter.