Radiometry

Computer Vision I
CSE 252A
Lecture 5

Radiometry

- Read Chapter 4 of Ponce & Forsyth
- Solid Angle
- Irradiance
- Radiance
- BRDF
- Lambertian/Phong BRDF

Measuring Angle

- The solid angle subtended by an object from a point P is the area of the projection of the object onto the unit sphere centered at P.
- Measured in steradians, sr
- Definition is analogous to projected angle in 2D
- If I’m at P, and I look out, solid angle tells me how much of my view is filled with an object

Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
- The solid angle subtended by a patch area dA is given by:

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

Radiance

- Power is energy per unit time
- Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle
- Symbol: \( L(x, \theta, \phi) \)
- Units: watts per square meter per steradian: \( \text{W} / (\text{m}^2 \text{sr}) \)

Radiance transfer

What is the power received by a small area dA2 at distance r from a small emitting area dA1?

From definition of radiance

\[ L = \frac{P}{(dA \cos \theta) d\omega} \]

From definition of solid angle

\[ d\omega = \frac{dA \cos \theta}{r^2} \]

\[ P = L dA \cos \theta d\omega, \quad \theta, \phi \]

\[ = \frac{L}{r^2} dA dA \cos \theta \cos \theta \]
**Irradiance**

- How much light is arriving at a surface?
- Sensible unit is \( \text{Irradiance} \)
- This is a function of incoming angle.
- A surface experiencing radiance \( L(x, \theta, \phi) \) coming in from solid angle \( d\omega \) experiences irradiance:

\[
E(x) = L(x, \theta, \phi) \cos \theta d\omega
\]

- Crucial property: Total power arriving at the surface is given by adding irradiance over all incoming angles. Total power is

\[
P(x) = \int_0^{2\pi} \int_0^{\pi/2} L(x, \theta, \phi) \cos \theta \sin \theta \sin \phi d\omega
\]

**Intermezzo: Camera’s sensor**

- Measured pixel intensity is a function of irradiance integrated over
  - pixel’s area
  - over a range of wavelengths
  - For some time

\[
I = \int \int \int E(x, y, \lambda, t) s(x, y) q(\lambda) d\omega dx d\lambda dt
\]

- Ideally, it’s linear to the radiance, but the camera response \( C(.) \) may not be linear

\[
I = C \left[ \int \int \int E(x, y, \lambda, t) s(x, y) q(\lambda) d\omega dx d\lambda dt \right]
\]

**Image sensor**

Two types:

1. **CCD**
2. **CMOS**

**CCD**

- separate photo sensor at regular positions
- no scanning
- charge-coupled devices (CCDs)

<table>
<thead>
<tr>
<th>Intercline transfer and Frame transfer</th>
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</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="CCD diagram" /></td>
</tr>
</tbody>
</table>

**CMOS**

- Each photo sensor has its own amplifier
- More noise (reduced by subtracting ‘black’ image)
- Lower sensitivity (lower fill rate)
- Uses standard CMOS technology
- Allows to put other components on chip
  - ‘Smart’ pixels

**CCD vs. CMOS**

- Mature technology
- Specific technology
- High production cost
- High power consumption
- Higher fill rate
- Blooming
- Sequential readout

- Recent technology
- Standard IC technology
- Cheap
- Low power
- Less sensitive
- Per pixel amplification
- Random pixel access
- Smart pixels
- On chip integration with other components
Color Cameras

We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel

... and X3

Prism color camera

Separate light in 3 beams using dichroic prism
Requires 3 sensors & precise alignment
Good color separation

Filter mosaic

Coat filter directly on sensor

Demosaicing (obtain full colour & full resolution image)

Filter wheel

Rotate multiple filters in front of lens
 Allows more than 3 colour bands

Prism vs. mosaic vs. wheel

<table>
<thead>
<tr>
<th>Approach</th>
<th># sensors</th>
<th>Separation</th>
<th>Cost</th>
<th>Frame rate</th>
<th>Artefacts</th>
<th>Bands</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism</td>
<td>3</td>
<td>High</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
<td>3</td>
</tr>
<tr>
<td>Mosaic</td>
<td>1</td>
<td>Average</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>3</td>
</tr>
<tr>
<td>Wheel</td>
<td>1</td>
<td>Good</td>
<td>Average</td>
<td>Low</td>
<td>Low</td>
<td>3 or more</td>
</tr>
</tbody>
</table>

Prism
- High-end cameras
- Scientific applications

Mosaic
- Low-end cameras
- Scientific applications

Wheel
- High-end cameras
- Scientific applications

new color CMOS sensor

Foveon’s X3

better image quality

smarter pixels
Example: Radiometry of thin lenses

\[
E = \frac{\pi d^2}{4z^2} \cos^4 \alpha \cos^4 \beta \cos \delta L
\]

- \( \pi d^2 \): Area of the lens
- \( L \): Emitted radiance
- \( d \): Lens diameter
- \( z \): Depth
- \( \alpha, \beta, \delta \): Angles from the optical axis

\[
\begin{align*}
\cos \alpha &= \frac{x}{r} \\
\cos \beta &= \frac{y}{r} \\
\cos \delta &= \frac{z}{r}
\end{align*}
\]

\[
\begin{align*}
\sin \alpha &= \frac{y}{z} \\
\sin \beta &= \frac{x}{z} \\
\sin \delta &= \frac{r}{z}
\end{align*}
\]

\[
\begin{align*}
E &= \frac{\pi d^2}{4z^2} \cos^4 \alpha \cos^4 \beta \cos \delta L \\
\end{align*}
\]

- \( E \): Image irradiance