Hw2 Solutions

11.2  (a) M=7×5×3=105. The range is [0, 104].

(b) M=105; m_3=7, m_2=5, m_1=3

\[ \hat{m}_3 = M/m_3 = 15, \quad \hat{m}_2 = M/m_2 = 21, \quad \hat{m}_1 = M/m_1 = 35 \]

|1/\hat{m}_3|_{m_3} = |1/15|_7 = 1, \quad |1/\hat{m}_2|_{m_2} = |1/21|_5 = 1, \quad |1/\hat{m}_1|_{m_1} = |1/35|_3 = 2

(2|3|2)RNS(7|5|3) = (2×1×15+3×1×21+2×1×35)_{105} = 23

(c)

\[ a_1 = x_1 = 2 \]

\[ a_2 = ((2|3|2)-(2|2|2)) \times |1/3|_5 = (0|1|-) \times 2 = (0|2|-) \]

\[ a_3 = ((0|2|-)-(2|2|2)) \times |1/5|_7 = (5|-|-) \times 3 = (1|-|-) \]

\( \Rightarrow \) (2|3|2)RNS(7|5|3) = (1|2|2)MRS(5|3)

(d) 20 = (6|0|2)RNS(7|5|3)

\[ a_3 = 20/15 = 1, \quad 20 \mod 15 = 5 \]

\[ a_2 = 5/3 = 1, \quad a_1 = 5 \mod 3 = 2 \]

20 = (1|1|2)MRS(5|3)

11.6  35 = (0|2|1)RNS(7|3|2)

(1/5|_3) = 3, \quad (1/5|_3) = 2, \quad (1/5|_2) = 1

35/5 = (0|2|1)RNS(7|3|2) \times (3|2|1)RNS(7|3|2) = (0|1|1)RNS(7|3|2) = 7

Because (1/7|_7) doesn’t exist, so 35/7 is not easy to be calculated.

34/5 is not an integer number that can be represented by RNS.

The conditions of performing division in RNS are: 1) the divisor is not a multiple of a base number; 2) the result is integer.
If the shifted-out bit is 0, the number is multiplied by 2.

<table>
<thead>
<tr>
<th>C(X)</th>
<th>C(X')</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

If the shifted-out bit is 1, the number is multiplied by 2, then decreased by $2^n$. When $n=5$:

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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

For example, $X=01101$, $C(X)=|X|=1$.

1st shift: $X'=11010$ $C(X')=2$

2nd shift: $X'=10100$ $C(X')=2$

3rd shift: $X'=01000$ $C(X')=2$

4th shift: $X'=10000$ $C(X')=1$

5th shift: $X'=00000$ $C(X')=0$