Photometric Stereo Recap

Computer Vision I
CSE252A
Lecture 8a

Announcements

- HW1 due on 2/4 (wed, not Tuesday.
- HW2 will be posted shortly, written assignment.
- Class web board available at:
  – http://www.etalonsoft.com/cse252a
  Thanks Louka.

Coordinate system

Photometric Stereo

\[ f(x, y) = (x, y, f(x, y)) \]

Tangent vectors:

\[ \mathbf{T} \]

Normal vector

\[ \mathbf{n} = \frac{\partial f}{\partial y} \times \frac{\partial f}{\partial x} \]

Reflectance map

For known BRDF, fix light source direction/strength and projection direction
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have \[ \mathbf{E}(p, q) \] which is known as the reflectance map

\[ \mathbf{n} = (p, q, -1)^T \]
Reflectance Map of Lambertian Surface

What does the value of one pixel in one image tell us?
It constrains normal to a curve

Isophote
Normal lies on this curve

Two Light Sources
Two reflectance maps

Third image would disambiguate match

Recovering the surface \( f(x,y) \)

Many methods: Simplest approach
1. From estimate \( \mathbf{n} = (n_x, n_y, n_z) \), \( p = n_x / n_z \), \( q = n_y / n_z \)
2. Integrate \( p = df/dx \) along a row \((x,0)\) to get \( f(x,0) \)
3. Then integrate \( q = df/dy \) along each column
   starting with value of first row

\( f(x,0) \)

What might go wrong?

• Height \( z(x,y) \) is obtained by integration along a curve
  from \((x_0, y_0)\).
  \[
  z(x, y) = z(x_0, y_0) + \int_{(x_0, y_0)}^{(x, y)} (pdx + qdy)
  \]

• If one integrates the derivative field along any closed curve,
  on expects to get back to the starting value.
• Might not happen because of noisy estimates of \((p, q)\)

Integrability

If \( f(x,y) \) is the height function, we expect that

\[
\frac{\partial f}{\partial y} = \frac{\partial f}{\partial x}
\]

In terms of estimated gradient space \((p,q)\), this means:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]

But since \( p \) and \( q \) were estimated
independently at each point as intersection
of curves on three reflectance maps,
equality is not going to exactly hold

II. Photometric Stereo:
Lambertian Surface,
Known Lighting
Lambertian Surface

At image location $(u,v)$, the intensity of a pixel $x(u,v)$ is:
\[ e(u,v) = [a(u,v) \cdot n(u,v)] \cdot [s_0^T \cdot s] \]
where
- $a(u,v)$ is the albedo of the surface projecting to $(u,v)$.
- $n(u,v)$ is the direction of the surface normal.
- $s_0$ is the light source intensity.
- $s$ is the direction to the light source.

Lambertian Photometric stereo

- If the light sources $s_1$, $s_2$, and $s_3$ are known, then we can recover $b$ from as few as three images. (Photometric Stereo: Silver 80, Woodham 81).

\[ [e_1 \ e_2 \ e_3] = b^T [s_1 \ s_2 \ s_3] \]

- i.e., we measure $e_1$, $e_2$, and $e_3$, and we know $s_1$, $s_2$, and $s_3$. We can then solve for $b$ by solving a linear system.

\[ b^T = [e_1 \ e_2 \ e_3] [s_1 \ s_2 \ s_3]^{-1} \]

- Normal is: $n = b/|b|$, albedo is: $|b|$

What if we have more than 3 Images?
Linear Least Squares

\[ [e_1 \ e_2 \ e_3] = b^T [s_1 \ s_2 \ s_3] \]

Let the residual be $r = e - Sb$

Rewrite as $e = Sb$

where
- $e$ is $n$ by 1
- $b$ is $3$ by 1
- $S$ is $n$ by 3

Squaring this:

\[ r^T r = e^T e - 2b^T S e + b^T S^T S b \]

\[ (r^T r)_{xx} = 0 \]

i.e., we measure $e_1$, $e_2$, and $e_3$, and we know $s_1$, $s_2$, and $s_3$. We can then solve for $b$ by solving a linear system.

Solving for $b$ gives

\[ b = (S^T S)^{-1} S^T e \]

Input Images

Recovered albedo

Recovered normal field
Surface recovered by integration