Lighting and Photometric Stereo

Computer Vision I
CSE252A
Lecture 7

• Read section 5.4, Ponce & Forsyth, Illumination Cone Paper
• HW1 posted to web site -- due 2/4/04
• Photometric stereo – Lambertian surfaces
• Illumination cones

Point source

• small, distant sphere radius \( \varepsilon \) and exitance \( E \), which is far away subtends solid angle of about

\[
\frac{\pi \varepsilon^2}{d^2}
\]

Diffuse lighting at infinity: Spherical Harmonics

Green: Positive
Blue: Negative

Radiance properties

• In free space, radiance is constant as it propagates along a ray
  – Derived from conservation of flux
  – Fundamental in Light Transport.

\[
\begin{align*}
\frac{d\Phi_i}{dA} &= \frac{L_i d\omega_i dA_i}{dA} = \frac{L_i d\omega_i}{dA} = \frac{d\Phi_i}{dA} \\
\frac{d\omega_i}{dA} &= \frac{dA_i}{r^2} \\
\frac{d\omega_i}{dA} &= \frac{dA_i}{r^2} \\
\frac{d\omega_i dA_i}{dA^2} &= \frac{dA_i}{r^2} = \frac{dA_i}{dA}
\end{align*}
\]

\[ L_i \cdot \frac{dA_i}{r^2} = \frac{dA_i}{dA} \]

\[ L_i = L_2 \]
Light Field/Lumigraph Main Idea

• In free space, the 5-D plenoptic function can be reduced to a 4-D function (radiances) on the space of light rays.
• Camera images measure the radiance over a 2-D set – a 2-D subset of the 4-D light field.
• Rendered images are also a 2-D subset of the 4-D lumigraph.

Shadows cast by a point source

• A point that can’t see the source is in shadow
• For point sources, the geometry is simple

Area Source Shadows

1. Fully illuminated
2. Penumbra
3. Umbra (shadow)

Photometric Stereo

I. Photometric Stereo:
   General BRDF and Reflectance Map

• Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.
• Photometric stereo: Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.
Coordinate system

Surface: \( s(x, y) = (x, y, f(x, y)) \)

Tangent vectors:
\[
\begin{pmatrix}
\frac{\partial s}{\partial x} \\
\frac{\partial s}{\partial y}
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} \frac{\partial f}{\partial x} + \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix} \frac{\partial f}{\partial y}
\]

Normal vector
\[
n = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \mathbf{k}
\]

Reflectance map

For known BRDF, fix light source direction/strength and projection direction
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have \( E(p, q) \) which is known as the reflectance map

Example Reflectance Map:
Lambertian surface

Two Light Sources
Two reflectance maps

Third image would disambiguate match
Photometric stereo:
Step 1
1. Acquire three images with known lighting.
2. Using known lighting & BRDF, construct reflectance map for each image.
3. For each pixel location, find (p,q) as intersection of three curves.
4. This is the surface normal at a pixel. Over image, this is normal field.

Plastic Baby Doll: Normal Field

Next step:
Go from normal field to surface

Recovering the surface f(x,y)
Many methods: Simplest approach
1. From estimate \( \mathbf{n} = (n_x, n_y, n_z) \), \( p = n_x / n_z \), \( q = n_y / n_z \)
2. Integrate \( p = df/dx \) along a row \((x,0)\) to get \( f(x,0) \)
3. Then integrate \( q = df/dy \) along each column starting with value of first row

What might go wrong?
• Height \( z(x,y) \) is obtained by integration along a curve from \((x_0, y_0)\):
  \[ z(x, y) = z(x_0, y_0) + \int_{(x_0, y_0)}^{(x, y)} (pdx + qdy) \]
• If one integrates the derivative field along any closed curve, one expects to get back to the starting value.
• Might not happen because of noisy estimates of \((p,q)\)

Integrability
If \( f(x,y) \) is the height function, we expect that
\[
\frac{\partial}{\partial y} \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}
\]
In terms of estimated gradient space \((p,q)\), this means:
\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]
But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold
Horn’s Method
[ “Robot Vision, B.K.P. Horn, 1986 ”]

• Formulate estimation of surface height \( z(x,y) \) from gradient field by minimizing cost functional:
\[
\int_{\text{Image}} (z_x - p)^2 + (z_y - q)^2 \, dx \, dy
\]
where \((p,q)\) are estimated components of the gradient while \(z_x\) and \(z_y\) are partial derivatives of best fit surface

• Solved using calculus of variations – iterative updating

• \( z(x,y) \) can be discrete or represented in terms of basis functions.

• Integrability is naturally satisfied.

II. Photometric Stereo:
Lambertian Surface, Known Lighting

Lambertian Surface

At image location \((u,v)\), the intensity of a pixel \( x(u,v) \) is:
\[
e(u,v) = [a(u,v) n(u,v)] \cdot [s_0 s] = b(u,v) \cdot s
\]
where

• \( a(u,v) \) is the albedo of the surface projecting to \((u,v)\).
• \( n(u,v) \) is the direction of the surface normal.
• \( s_0 \) is the light source intensity.
• \( s \) is the direction to the light source.

Lambertian Photometric stereo

• If the light sources \( s_1, s_2, \) and \( s_3 \) are known, then we can recover \( b \) from as few as three images. (Photometric Stereo: Silver 80, Woodham81).

\[
[e_1 \ e_2 \ e_3] = b^T [s_1 \ s_2 \ s_3]
\]

• i.e., we measure \( e_1, e_2, \) and \( e_3 \) and we know \( s_1, s_2, \) and \( s_3 \). We can then solve for \( b \) by solving a linear system.

\[
b^T = [e_1 \ e_2 \ e_3] [s_1 \ s_2 \ s_3]^{-1}
\]

• Normal is: \( n = b/|b| \), albedo is: \(|b|\)

What if we have more than 3 Images?
Linear Least Squares

\[
[e_1 \ e_2 \ e_3] = b^T [s_1 \ s_2 \ s_3]
\]

Let the residual be
\[
r = e - Sb
\]

Rewrite as
\[
e = Sb
\]

where

\[
e \text{ is } n \times 1
\]
\[
b \text{ is } 3 \times 1
\]
\[
S \text{ is } n \times 3
\]

Squaring this:
\[
r^2 = r^T r = (e - Sb)^T (e - Sb) = e^T e - 2S^T e + b^T S^T b
\]

\((r^2) = 0 \cdot \text{ zero derivative is a necessary condition for a minimum, or}
\[-2S^T e + 2S^T S b = 0;
\]

Solving for \( b \) gives
\[
b = (S^T S)^{-1} S^T e
\]

Input Images
Recovered albedo

Recovered normal field

Surface recovered by integration

Lambertian Photometric Stereo

Reconstruction with albedo map

Without the albedo map
Another person

No Albedo map

III. Photometric Stereo with unknown lighting and Lambertian surfaces

Covered in Illumination cone slides