Lighting and Photometric Stereo

Computer Vision I
CSE252A
Lecture 6

Lighting

• Read sections 5.2-5.4, Ponce & Forsyth
• HW1 will be posted to web site later today, due 2/4/04

• Light sources
• Global vs. local shading models & inter-reflection
• Photometric stereo

Radiance

• Power is energy per unit time
• Radiance: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle
• Symbol: \( L(x, \theta, \phi) \)
• Units: watts per square meter per steradian: \( \text{w/(m}^2\text{sr}) \)

\[
L = \frac{P}{(dA \cos \theta)d\omega}
\]

Irradiance

• How much light is arriving at a surface?
• Sensible unit is Irradiance
• Incident power per unit area not foreshortened
• This is a function of incoming angle.
• A surface experiencing radiance \( L(x, \theta, \phi) \) coming in from solid angle \( d\omega \) experiences irradiance:

\[
\int_{\Omega} L(x, \theta, \phi) \cos \theta \sin \theta d\omega
\]

Intermezzo: Camera’s sensor

• Measured pixel intensity is a function of irradiance integrated over
  – pixel’s area
  – over a range of wavelengths
  – For some time

\[
I = \int \int \int E(x, y, \lambda, t)s(x, y)g(\lambda)dydxdt
\]

• Ideally, it’s linear to the radiance, but the camera response \( C(\cdot) \) may not be linear

\[
I = C \left( \int \int \int E(x, y, \lambda, t)s(x, y)g(\lambda)dydxdt \right)
\]

Prism vs. mosaic vs. wheel

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<td>Scientific applications</td>
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New color CMOS sensor
Foveon’s X3

Better image quality
Smarter pixels

Example: Radiometry of thin lenses

Example: Radiometry of thin lenses

E = \frac{\pi}{4} \left( \frac{d^2}{z} \right) \cos^4 \alpha L

E: Image irradiance
L: emitted radiance
d: Lens diameter
z: depth
\alpha: Angle of patch from optical axis

Properties of BRDF’s

Isotropy: BRDF is a function of \theta_i, \theta_r, \Phi_i - \Phi_r

Helmholtz Reciprocity: symmetric function
\rho(\theta_i, \Phi_i, \theta_r, \Phi_r) = \rho(\theta_r, \Phi_r, \theta_i, \Phi_i)

BRDF Not Always Appropriate

http://graphics.stanford.edu/papers/bssrdf/
(Jensen, Marschner, Levoy, Hanrahan)
Reflectance models

- BRDF: 4-D function
- BTF: Bidirectional texture function 6-D function. Spatially varying BRDF
- BSSRDF 6-D function (or 8-D if spatially varying).

Light sources and shading

- How do we describe/represent light sources?
- One more definition: Exitance of a source is
  - the internally generated power radiated per unit area on the radiating surface
  - Similar to irradiance
- Common sources:
  1. Point sources
  2. Sources at infinity
  3. Line sources
  4. Area sources
  5. 4-D function

Point source

- small, distant sphere radius ε and exitance E, which is far away substends solid angle of about \( \pi \epsilon^2 / d^2 \)

Standard nearby point source model

\[ \rho_d(x) N(x) \cdot S(x) \]

\( N \) is the surface normal
\( \rho \) is diffuse (Lambertian) albedo
\( S \) is source vector - a vector from \( x \) to the source

Standard distant point source model

- Issue: nearby point source gets bigger if one gets closer
  - the sun doesn’t for any reasonable binding of closer
- Assume that all points in the model are close to each other with respect to the distance to the source. Then the source vector doesn’t vary much, and the distance doesn’t vary much either, and we can roll the constants together to get:

\[ \rho_d(x) N(x) \cdot S(x) / r(x)^2 \]

Line sources

radiosity due to line source varies with inverse distance, if the source is long enough
Area sources

- Examples: diffuser boxes, white walls.
- The radiosity at a point due to an area source is obtained by adding up the contribution over the section of view hemisphere subtended by the source – change variables and add up over the source
- See Forsyth & Ponce or a graphics text for details.

Diffuse lighting at infinity: Spherical Harmonics

\[ Y_{lm}(\theta, \phi) \]

Order

\[ l=0 \]
\[ l=1 \]
\[ l=2 \]

\[ m=-2 \quad m=-1 \quad m=0 \quad m=1 \quad m=2 \]

Green: Positive
Blue: Negative

(Borrowed from Ramamoorthi, Hanrahan, SIGGRAPH'01)

Images as a Collection of Rays

Conversely, the light emitted at a given point also is a function on a 2-D space (sphere)

Conversely, the set of light rays emitted from all points...

Radiance properties

- In free space, radiance is constant as it propagates along a ray
  - Derived from conservation of flux
  - Fundamental in Light Transport.

\[ d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2 \]

\[ d\omega_1 = d\omega_2 = \frac{dA_1}{r^2} = \frac{dA_2}{r^2} \]

\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \quad \therefore L_1 = L_2 \]

Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?
Light Field/Lumigraph Main Idea

- In free space, the 5-D plenoptic function can be reduced to a 4-D function (radiances) on the space of light rays.
- Camera images measure the radiance over a 2-D set – a 2-D subset of the 4-D light field.
- Rendered images are also a 2-D subset of the 4-D lumigraph.

Shadows cast by a point source

- A point that can’t see the source is in shadow
- For point sources, the geometry is simple

Area Source Shadows

1. Fully illuminated
2. Penumbra
3. Umbra (shadow)

Shading models

Local shading model
- Surface has incident radiance due only to sources visible at each point
- Advantages:
  - often easy to manipulate, expressions easy
  - supports quite simple theories of how shape information can be extracted from shading
- Used in vision & real time graphics

Global shading model
- Surface radiosity is due to radiance reflected from other surfaces as well as from surfaces
- Advantages:
  - usually very accurate
- Disadvantage:
  - extremely difficult to infer anything from shading values
- Rarely used in vision, often in photorealistic graphics

At the top, geometry of a gutter with triangular cross-section; below, predicted radiosity solutions, scaled to lie on top of each other, for different albedos of the geometry. When albedo is close to zero, shading follows a local model; when it is close to one, there are substantial reflexes.

Irradiance observed in an image of this geometry for a white gutter.
Photometric Stereo

- Shape-from-shading: Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

- Photometric stereo: Single viewpoint, multiple images under different lighting.
  1. Arbitrary known BRDF
  2. Lambertian BRDF, known lighting
  3. Lambertian BRDF, unknown lighting.

Photometric Stereo: General BRDF and Reflectance Map

An example of photometric stereo
Coordinate system

1. Orthographic projection
2. Parameterize image plane by (x,y)
3. Recover height function f(x,y)
   - Monge patch
     Surface parameterized as: \( (x,y) \rightarrow (x,y,f(x,y)) \)

Surface: \( s(x,y) = (x,y,f(x,y)) \)
Tangent vectors:
\[
\begin{bmatrix}
\frac{\partial s}{\partial x} \\
\frac{\partial s}{\partial y}
\end{bmatrix} = \begin{bmatrix}
1 \\
0
\end{bmatrix}, \begin{bmatrix}
0 \\
1
\end{bmatrix}, \begin{bmatrix}
1 \\
1
\end{bmatrix}
\]

Normal vector:
\[
\mathbf{n} = \frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y} = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \right)
\]

Gradient Space: \((p,q)\)
\[
p = \frac{\partial f}{\partial x}, \quad q = \frac{\partial f}{\partial y}
\]

Image Formation

For a given point \( p \) on the surface, the image irradiance \( E(x,y) \) is a function of
1. The BRDF at \( p \)
2. The surface normal at \( p \)
3. The direction of the light source

Example Reflectance Map:
Lambertian surface

Now fix BRDF and fix light source direction/strength.
1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have \( E(p,q) \).
What does the value of one pixel in one image tell us?
It constrains normal to a curve

Two Light Sources
Two reflectance maps

Third image would disambiguate match

Photometric stereo:
Step1
1. Acquire three images with known lighting.
2. Using known lighting & BRDF, construct reflectance map for each image.
3. For each pixel location, find (p,q) as intersection of three curves.
4. This is the surface normal at a pixel. Over image, this is normal field.
Plastic Baby Doll: Normal Field

Next step:
Go from normal field to surface

Recovering the surface \( f(x,y) \)
Many methods: Simplest
1. Integrate \( p = df/dx \) along a row \((x,0)\) to get \( f(x,0) \)
2. Then integrate \( q = df/dy \) along each column starting with value of first row

\[
\begin{align*}
\frac{\partial}{\partial y} f & = \frac{\partial f}{\partial x} \\
\frac{\partial}{\partial x} f & = \frac{\partial f}{\partial y}
\end{align*}
\]

In terms of estimated gradient space \((p,q)\), this means:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]

But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold

What might go wrong?
Integrability. If \( f(x,y) \) is the height function, we expect that

\[
\frac{\partial}{\partial y} f = \frac{\partial f}{\partial x}
\]

\[
\frac{\partial}{\partial x} f = \frac{\partial f}{\partial y}
\]

In terms of estimated gradient space \((p,q)\), this means:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]

But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold