Announcements

- HW3: Stereo, On web page, Due 3/2/04
- Final Exam: Friday, March 19, 11:30-2:30

- Today:
  – Stereo using Dynamic Programming
  – Motion

Slide Window to different disparities to find best match

Best Match amounts to minimizing (or maximizing some) Match Metric

Match Metric Summary

<table>
<thead>
<tr>
<th>MATCH METRIC</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized Cross-Correlation (NCC)</td>
<td>$\frac{\sum (I(x,y) \cdot \hat{I}(x,y))}{\sqrt{\sum I(x,y)^2 \cdot \sum \hat{I}(x,y)^2}}$</td>
</tr>
<tr>
<td>Sum of Squared Differences (SSD)</td>
<td>$\sum (I(x,y) - \hat{I}(x,y))^2$</td>
</tr>
<tr>
<td>Normalized SSD</td>
<td>$\frac{\sum (I(x,y) - \hat{I}(x,y))^2}{\sum I(x,y)^2}$</td>
</tr>
<tr>
<td>Sum of Absolute Differences (SAD)</td>
<td>$\sum \left</td>
</tr>
<tr>
<td>Zero Mean SAD</td>
<td>$\sum \left</td>
</tr>
<tr>
<td>Census</td>
<td>$\sum (I(x,y) - \hat{I}(x,y))$</td>
</tr>
</tbody>
</table>

These two are actually the same

Window size

- Effect of window size
  • Better results with adaptive window

Problem of Occlusion

Better results with adaptive window

(Seitz)
Stereo Constraints

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Brief Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D Epipolar Search</td>
<td>Arbitrary images of the same scene may be rectified based on epipolar geometry.</td>
</tr>
<tr>
<td>Monotonic Ordering</td>
<td>Points along an epipolar scanline appear in the same order in both images.</td>
</tr>
<tr>
<td>Image Brightness Constancy</td>
<td>Corresponding points in stereo images are the same brightness.</td>
</tr>
<tr>
<td>Match Uniqueness</td>
<td>For every point in one image, there is at most one corresponding point in the other</td>
</tr>
<tr>
<td>Disparity Continuity</td>
<td>Disparities vary smoothly over most of the image.</td>
</tr>
<tr>
<td>Disparity Limit</td>
<td>The search space may be limited by the disparity range.</td>
</tr>
<tr>
<td>Fronto-Parallel Surfaces</td>
<td>Objects have fronto-parallel surfaces.</td>
</tr>
<tr>
<td>Feature Similarity</td>
<td>Corresponding features must be similar.</td>
</tr>
</tbody>
</table>

Stereo matching

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Summary measures (SSD or NCC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epipolar</td>
<td>Optimal path (dynamic programming)</td>
</tr>
<tr>
<td>Ordering</td>
<td>Trade-off:</td>
</tr>
<tr>
<td>Uniqueness</td>
<td>• Matching cost (data)</td>
</tr>
<tr>
<td>Disparity limit</td>
<td>• Discontinuities (prior)</td>
</tr>
<tr>
<td>Disparity gradient limit</td>
<td></td>
</tr>
</tbody>
</table>

Dynamic Programming

- Efficient algorithm for solving sequential decision (optimal path) problems.

Suppose cost can be decomposed into stages:
\[ \Pi_j = \text{Cost of going from state } i \text{ to state } j \]

Principle of Optimality for an n-stage assignment problem:
\[ C_i(j) = \min \{ \Pi_{i,j} + C_j(i) \} \]

Occluded Pixels

Dynamic programming yields the optimal path through grid. This is the best set of matches that satisfy the ordering constraint.

Every pixel on each scanline will be labeled as matching, or occluded.
Dynamic Programming

\[ C(i,j) = \min \{ C_{i-1,j-1} + \text{match-cost of pixel L(i) & R(i)}, \ C_{i-1,j} + \text{occlusion-penalty}, \ C(i,j-1) + \text{occlusion-penalty} \} \]

\[ b(i,j) = \arg \min \{ C_{i-1,j-1}, C_{i-1,j}, C(i,j-1) \} \]

\[ b(i,j) \text{ gives previous state along minimum cost path} \]

Back tracking to get optimal path

Stereo Matching with Dynamic Programming

\[ C(i,j) \text{ is minimum of} \]

1. \[ C(i-1,j-1) + \text{match-cost of pixel L(i) & R(i)} \]
2. \[ C(i-1,j) + \text{occlusion-penalty} \]
3. \[ C(i,j-1) + \text{occlusion-penalty} \]
**Stereo Matching with Dynamic Programming**

Scan across grid computing optimal cost for each node given its upper-left neighbors.

Scan across grid computing optimal cost for each node given its upper-left neighbors.

Scan across grid computing optimal cost for each node given its upper-left neighbors.

Scan across grid computing optimal cost for each node given its upper-left neighbors.

Backtrack from the terminal to get the optimal path.

Backtrack from the terminal to get the optimal path.

Backtrack from the terminal to get the optimal path.

Backtrack from the terminal to get the optimal path.

Some Challenges & Problems

- Photometric issues:
  - specularities
  - strongly non-Lambertian BRDF’s
- Surface structure
  - lack of texture
  - repeating texture within horopter bracket
- Geometric ambiguities
  - as surfaces turn away, difficult to get accurate reconstruction (affine approximate can help)
  - at the occluding contour, likelihood of good match but incorrect reconstruction

Many variations

- Subpixel interpolation
- Probabilistic framework
- Creases
- Occlusion penalties
Variations on Binocular Stereo

1. Trinocular Stereopsis
2. Helmholtz Reciprocity Stereopsis

Helmholtz reciprocity

\[ p(\theta_{in}, \phi_{in}; \theta_{out}, \phi_{out}) = p(\theta_{out}, \phi_{out}; \theta_{in}, \phi_{in}) \]

[Helmholtz, 1910], [Minnaert, 1941], [Nicodemus et al., 1977]

Point Source Illumination

\[ i = p(\psi, \phi, \hat{n}, \psi, \phi, \hat{n}) \]

Matching Constraint

\[ \hat{n} = 0 \]

computed from geometric calibration

Using Multiple Helmholtz Stereo Pairs

\[ \hat{n} = 0 \]

Multiple views (at least three pairs) yield a matrix constraint equation.
Matrix must be Rank 2.
Search for depth where rank constraint is satisfied.
Finding the Normal at each point

\[
\hat{n} = 0
\]

Additionally, the surface normal \( \hat{n} \) must lie in the kernel of the matrix.

Experimental Setup

Disparity and Normal Field

Bulldog: Disparity

Bulldog: Normal Field
Plastic Baby Doll: Normal Field

Plastic Baby Doll: Disparities

Surface after integrating normal field

Renderings of Reconstruction

Comparison to other methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Anisotropic Reflections</th>
<th>Surface Illumination Recovered</th>
<th>Recovering Occluded Indirect</th>
<th>Active/Passive</th>
<th>Depth Discontinuities Recovered</th>
<th>Handles/No Occlusion</th>
<th>Robust to Cast Shadows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photometric stereopsis</td>
<td></td>
<td></td>
<td></td>
<td>Yes</td>
<td>No</td>
<td>NA</td>
<td>No</td>
</tr>
<tr>
<td>Radiometric stereopsis</td>
<td></td>
<td></td>
<td></td>
<td>Sometimes</td>
<td>Sometimes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Hybridized stereopsis</td>
<td></td>
<td></td>
<td></td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

More on stereo ...

The Middlebury Stereo Vision Research Page
http://cat.middlebury.edu/stereo/

Recommended reading

D. Scharstein and R. Szeliski.