Announcements

• HW3: Stereo, On web page Due 2/26/04
• Final Exam: Friday, March 19, 11:30-2:30
• Today: More stereo

Need for correspondence

Two Approaches

• A) From each image, process “monocular” image to obtain cues.
• B) Establish correspondence between cues.
• Directly compare image regions between the two images.

Random Dot Stereograms

A Cooperative Model (Marr and Poggio, 1976)
Epipolar Constraint

- Potential matches for $p$ have to lie on the corresponding epipolar line $l'$.
- Potential matches for $p'$ have to lie on the corresponding epipolar line $l$.

Epipolar Geometry

- Baseline
- Epipole
- Epipolar Line

Family of epipolar Planes

(standard approach)

Properties of the Essential Matrix

- $p' \mathcal{E} p = 0$ with $\mathcal{E} = [t, R]$. 
- $E \mathbf{p}'$ is the epipolar line associated with $p'$.
- $E^T \mathbf{p}$ is the epipolar line associated with $p$.
- $E e' = 0$ and $E e = 0$.
- $E$ is singular.
- $E$ has two equal non-zero singular values (Huang and Faugeras, 1989).

The Eight-Point Algorithm (Longuet-Higgins, 1981)

Much more on multi-view in CSE252B!!!

\[
\begin{pmatrix}
F_{11} & F_{12} & F_{13} \\
F_{21} & F_{22} & F_{23} \\
F_{31} & F_{32} & F_{33}
\end{pmatrix}
\begin{pmatrix}
\mathbf{u}' \\
1
\end{pmatrix} = 0
\]

Set $F_{33}$ to 1

Minimize:

\[
\sum_{i=1}^{n} (p_i^T F p_i)^2
\]

under the constraint $|F|^2 = 1$. 

The Essential Matrix

(Longuet-Higgins, 1981)
Example: converging cameras

Example: motion parallel with image plane

Example: forward motion

Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Input Images

Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.
Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Rectified Images
See Section 7.3.7 for specific method

Features on same epipolar line

Mobi: Stereo-based navigation

Epipolar correspondence

Symbolic Map

Multiple Interpretations
Each feature on left epipolar line match one and only one feature on right epipolar line.
Multiple Interpretations

Each feature on left epipolar line match one and only one feature on right epipolar line.

Correspondence: Photometric constraint

- Same world point has same intensity in both images (Constant Brightness Constraint)
  - Lambertian fronto-parallel
  - Issues:
    - Noise
    - Specularity
    - Foreshortening

Using epipolar & constant Brightness constraints for stereo matching

For each epipolar line
For each pixel in the left image
  - compare with every pixel on same epipolar line in right image
  - pick pixel with minimum match cost
  - This will never work, so:

Improvement: match windows

(Camps)

Comparing Windows:

\[
SSD = \sum_{i, n \in R} (f(i, j) - g(i, j))^2
\]

\[
C_{fp} = \sum_{i, n \in R} f(i, j)g(i, j)
\]

Most popular

For each window, match to closest window on epipolar line in other image.
### Match Metric Summary

<table>
<thead>
<tr>
<th>MATCH METRIC</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalized Cross Correlation (NCC)</td>
<td>( \frac{\sum (u,v) (I_1(u,v) - I_2(u,v))^2}{\sum (u,v) (I_1(u,v) - \bar{I_1})^2} )</td>
</tr>
<tr>
<td>Sum of Squared Differences (SSD)</td>
<td>( \sum (u,v) (I_1(u,v) - I_2(u,v))^2 )</td>
</tr>
<tr>
<td>Normalized SSD</td>
<td>( \frac{\sum (u,v) (I_1(u,v) - I_2(u,v))^2}{\sum (u,v) (I_1(u,v) - \bar{I_1})^2} )</td>
</tr>
<tr>
<td>Sum of Absolute Differences (SAD)</td>
<td>( \sum (u,v)</td>
</tr>
<tr>
<td>Zero Mean SAD</td>
<td>( \sum (u,v) (I_1(u,v) - \bar{I_1})(I_2(u,v) - \bar{I_2}) )</td>
</tr>
<tr>
<td>Rank</td>
<td>( \sum (u,v)</td>
</tr>
<tr>
<td>Census</td>
<td>( \sum (u,v) \delta(I_1(u,v) - I_2(u,v)) )</td>
</tr>
</tbody>
</table>

*These two are actually the same*

### Correspondence Search Algorithm

(simple version for Cross Correlation)

For \( i = 1 \) to \( n_{rows} \)
for \( j = 1 \) to \( n_{cols} \)
best\((i,j)\) = -1
for \( k = \text{mindisparity} \) to \( \text{maxdisparity} \)
\( c = \text{ACC}(I_1(i,j), I_2(i,j+k), \text{winsize}) \)
if \( c > \text{best}(i,j) \)
best\((i,j)\) = \( c \)
disparities\((i,j)\) = \( k \)
end
end
end
\( O(n_{rows} \times n_{cols} \times \text{disparities} \times \text{winx} \times \text{winy}) \)

### Stereo results

– Data from University of Tsukuba

Scene

Ground truth

(Seitz)

### Results with window correlation

Window-based matching (best window size)

Ground truth

(Seitz)

### Results with better method

State of the art method

Ground truth

(Seitz)

### Window size

- Effect of window size

Better results with adaptive window

Stereo Constraints

<table>
<thead>
<tr>
<th>CONSTRAINT</th>
<th>BRIEF DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D Epipolar Search</td>
<td>Arbitrary images of the same scene may be rectified based on epipolar geometry such that stereo matches lie along one-dimensional scanlines. This reduces the computational complexity and also reduces the likelihood of false matches.</td>
</tr>
<tr>
<td>Monotonic Ordering</td>
<td>For every pixel in one stereo image, there is at most one corresponding pixel in the other image.</td>
</tr>
<tr>
<td>Image Brightness Constancy</td>
<td>Corresponding pixels have roughly the same brightness.</td>
</tr>
<tr>
<td>Match Uniqueness</td>
<td>Corresponding features must be similar (e.g., edges must have roughly the same length and orientation).</td>
</tr>
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<td>Disparity Continuity</td>
<td>The search space may be reduced significantly by limiting the disparity range, reducing both computational complexity and the likelihood of false matches.</td>
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<td>Fronto-Parallel Surfaces</td>
<td>Corresponding feature groupings and their connectivity must be consistent.</td>
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<td>Feature Similarity</td>
<td>Corresponding feature groupings and their connectivity must be consistent.</td>
</tr>
<tr>
<td>Structural Grouping</td>
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</tr>
</tbody>
</table>

Problem of Occlusion

Stereo Matching using Dynamic Programming

Stereo Matching with Dynamic Programming

Dynamic programming yields the optimal path through grid. This is the best set of matches that satisfy the ordering constraint. Every pixel on each scanline will be labeled as matching, or occluded.