1. Problem 3.4

Initial state \((y_1, y_2) = (0, 0)\) fault \(a \rightarrow 1 \rightarrow \)}

\[
\begin{align*}
(0, 0) & \overset{x=1}{\longrightarrow} y_1 = \overline{D}, y_2 = 1 \quad Z = 0 \\
(\overline{D}, 1) & \overset{x=0}{\longrightarrow} y_1 = D, y_2 = D \quad Z = 0 \\
(D, D) & \overset{x=1}{\longrightarrow} y_1 = 1, y_2 = \overline{D} \quad Z = D
\end{align*}
\]

\[T = 101, \quad Z = 00D\]
2. Problem 3.5

![Logic Diagram]

**Fault 2 s-a-0**

(a) Initial state \((1, 1) = (\overline{y_1}, y_2)\). To test for 2 s-a-0, goal state is \(\overline{y_2} = 1, y_1 = 0\).

\[
egin{align*}
(1,1) & \xrightarrow{x=0} (1,0), z = 0 \\
(1,0) & \xrightarrow{x=1} (0,0), z = 0 \\
(0,0) & \xrightarrow{x=1} (0,0), z = 0
\end{align*}
\]

\[T = 011, \ z = 000\]

(b) Fault 2 s-a-0. Initial state \((y_1, y_2) = (u, u)\)

\[
egin{align*}
(u, u) & \xrightarrow{x=0} (u, u), z = u \\
& \xrightarrow{x=1} (u, u), z = u
\end{align*}
\]

No test exists.

Circuit can not be initialized
A decoder is a combinational circuit containing \( n \) inputs and \( 2^n \) outputs, labeled respectively \( (x_0, x_1, \ldots, x_{n-1}) \) and \( (z_0, z_1, \ldots, z_{2^n-1}) \). The circuit functions as follows: If \( (a_0, a_1, \ldots, a_{n-1}) \) is a binary input vector, representing the integer \( k \), then \( z_k = 1 \) and \( z_j = 0 \) for all \( j \neq k \). Based upon the functional operation of this device, determine a minimal set of tests which will detect any single fault on an input or output line of this device.

To detect \( z_k \) for any \( k \), \( 0 \leq k \leq 2^{n-1} \), apply an input corresponding to integer \( k \).

Doing this for all \( k \), \( 0 \leq k \leq 2^{n-1} \), requires \( 2^n \) tests (exhaustive testing - all possible inputs must be applied).
2.19 A multiplexer is a combinational circuit $C$ having the structure shown in Figure 2.60.

![Figure 2.60](image)

If $(a_0, a_1, \ldots, a_{n-1})$ is an input vector, representing the integer $k$, applied to $(x_0, y_3, \ldots, x_{n-1})$, then $z = x_k$, where $0 \leq k \leq 2^n - 1$. That is, input line $x_k$ is connected to the output $z$. Based upon the functional operation of this device, determine a minimal set of tests which will detect any single stuck fault on an input or output line of this device.

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4. Problem # 2.19

For each of the $2^n$ values of $(y_0, \ldots, y_{n-1})$ corresponding to $x_0$ input, apply 2 tests, one with $x_2 = 0$, $x_y = 1$ for all $y$, the other with $x_2 = 1$, $x_y = 0$ for all $y \neq 1$. These $2 \times 2^n = 2^{n+1}$ tests detect any $s.a.$ fault on $x$ inputs, $y$ inputs or $z$ output.

This requires $2^{n+1}$ tests out of a possible $2^{(n+2)}$ input combinations.

$$(2^{n+1}/2^{2^n}) = \frac{1}{2^{n-1}}$$

For $m=2$ this is $1/8$.

For $m=2$ there are 6 inputs $(x_0, x_1, x_3, y_0, y_1)$.

The 8 tests are

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 \\
\end{array}
\]
3. Problem 3.20

3 state machine

\[
\begin{array}{ccccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\
X: & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
Z: & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & a_3 a_4 \\
\end{array}
\]

Notice 3 different responses to input sequence 00 at \(t=1\), 10 at \(t=2\), 00 at \(t=3\). Let state at \(t=1\) be A, state at \(t=2\) is B, state at \(t=3\) is C. Then \(N(A,0) = B\), \(N(B,0) = C\), \(Z(B,0) = 0\), \(N(C,0) = A\) or B, \(Z(C,0) = 1\). The state at \(t=6\) must be C (since only state C has \(Z=1\) for \(x=1\)). The state at \(t=5\) cannot be C since \(N(C,0) \neq C\). Therefore \(A_1 = 0\). Therefore the state at \(t=4\) is A and \(N(C,0) = A\).

Therefore the state at \(t=7\) is A and the state at \(t=8\) is C. Hence \(N(A,1) = C\), \(Z(A,1) = 0\) and \(A_2 = 0\).

The state at \(t=10\) is B and the state at \(t=11\) is C. Hence \(N(B,1) = C\), \(Z(B,1) = 0\).
The values of $a_3, a_4$ must be $a_3 = a_4 = 0$ and the state at $t = 14$ is C. To determine the values of $N(C, 1)$ and $Z(C, 1)$ we add the following subsequence.

\[ \begin{array}{c|cc} \hline x & 0 & 1 \\ \hline A & B, 0 & C, 0 \\ B & C, 0 & C, 0 \\ C & A, 1 & \hline \end{array} \]

$X \ 1 \ 0 \ 0 \ 0$

$a_5 \ a_6 \ a_7$

The values of $Z$ at $t = 15$ and $16$ uniquely determine the state at $t = 15$ and hence the value of $N(C, 1)$. The output at $t = 14$ is $Z(C, 1)$.

\[ a_6 \ a_7 \ 00 \Rightarrow N(C, 1) = A \]
\[ a_6 \ a_7 \ 01 \Rightarrow N(C, 1) = B \]
\[ a_6 \ a_7 \ 10 \Rightarrow N(C, 1) = C \]

$Z(C, 1) = 9$