Problem 2.2

\[
\begin{array}{c}
\text{Normal} \\
\{ \begin{array}{c}
G_5 \quad G_6 \quad G_7 \\
\downarrow \quad \downarrow \\
1 \quad x \quad 1 \\
\_ \quad \_ \quad \_ \\
0 \quad 0 \quad 0 \\
\end{array}
\} \\
\text{Faulty} \\
\{ \begin{array}{c}
G_5 \quad G_6 \quad G_7 \\
\downarrow \quad \downarrow \\
1 \quad 1 \quad 1 \\
\_ \quad \_ \quad \_ \\
0 \quad 0 \quad 0 \\
\end{array}
\}
\end{array}
\]

Primitive deceses of F

\[
\begin{array}{c}
G_5 \quad G_6 \quad G_7 \\
\downarrow \quad \downarrow \\
1 \quad 0 \quad D \\
\_ \quad \_ \quad \_ \\
0 \quad 0 \quad D \\
\end{array}
\]

Hence the only non-tests must produce \( G_5 = G_6 = 1 \)

\( G_5 = 1 \) requires \( B = 1 \) and \( G_2 = 1 \)

\( G_6 = 1 \) requires \( G_3 = 1 \) and \( G_4 = 1 \)

\( G_4 = 1 \) requires \( G_2 = 0 \)

Contradiction \( G_2 = 1 \), and \( G_2 = 0 \)

Hence there is no input which does not detect the fault (i.e. all inputs are tests).
Problem 2.3

First we will try to derive tests which sensitise
the fault A 5-a-1.

This requires A = 0. To propagate through G2

→ G1 = 0 → (C = 0 or E = 0)

We can then propagate through G5 by B = 1.

If E = 0 then G3 = 0 and G6 = 0 and the
fault is propagated to the output.

This leads to the test \( \overline{ABE} = (0, 1, 0, 0) \).

Alternatively if B = 0 the A 5-a-1 fault is propagated
through G6 leading to an error signal D on output
of G5 (D acts as a-2 \( \Rightarrow D \)) and the fault propagates
to the output leading to the test \( \overline{AB} (\overline{E} + E) \)

= (0, 0, 0, 0), (0, 0, 0, 0).

We can propagate the A 5-a-1 fault through both G5 and G6 (by B = 1, E = 1) but this leads to
\( D + \overline{D} = 1 \) at G7 (errors cancel due to opposite polarity).

By setting A = 1 we desensitize the fault A 5-a-1.

The fault G5 5-a-0 is sensitized by G4 = G3 = 1.

However G4 = 1 implies G2 = 0 \( \Rightarrow A = 0 \), which is a contradiction.

Hence no tests have the value A = 1.

(3) Problem 2.4

(a) to detect A 5-a-0

A = 1, B = 1 (to propagate through G1)

G2 = 0, G3 = 0 (to propagate through G4)

G2 = 0 \( \Rightarrow \) requires D = 1, B = 1. A B E D is only test
G2 = 0 \( \Rightarrow \) requires C = 0
(b) Detect b s-a-1
\[ B = 0, \quad A = 1 \quad (\text{to propagate through } G_1) \]

However \( B = 0 \) implies \( G_2 = 1 \)
which implies \( Z = 1 \).
Hence fault is not detectable.
(Note \( G_1 \) can be removed and replaced by \( A \) input to OR gate with no change in output function.

(c) Detect \( G_2 \) s-a-1
\[ G_2 = 0 \] requires \( B = 1, \quad D = 1 \).
To propagate through \( G_4 \) we require \( G_1 = G_3 = 0 \).
\( G_3 = 0 \) implies \( C = 0 \) (since \( D \) has already been set to 1)
\( G_1 = 0 \) implies \( A = 0 \) (since \( B \) is fixed)

Only test which detects \( G_2 \) s-a-1
is \( \overline{A}BCD \).

(d) Detect multiple fault \( b \) s-a-1, \( G_2 \) s-a-1.

(1) Sensitize \( G_2 \) fault by \( B = D = 1 \) \( \Rightarrow G_2 = \overline{D} \)
This desensitizes \( b \) s-a-1 fault,
hence \( \overline{A}BCD \) detects multiple fault.

(2) Sensitize \( b \) s-a-1 fault by \( B = 0 \)
This desensitizes \( G_2 \) fault by produced \( G_2 = 1 \)
but results in \( Z = 1 \) and does not detect multiple fault.

Hence only \( \overline{A}BCD \) detects multiple fault \( b \) s-a-1,
\[ G_2 \) s-a-1. \]
Problem 2.5

(a) Tests which detect A s-a-0
\[ A = 1, \quad B = 1 \text{ (to propagate through } G_1) \]
\[ G_2 = 0 \text{ (to propagate through } G_4) \Rightarrow B = 1 \text{ and } D = 1 \]
\[ G_3 = 0 \text{ (to propagate through } G_4) \Rightarrow C = 0 \text{ or } D = 0 \]

1. Test is \( A \overline{B} \overline{C} \overline{D} = T_\alpha \)

Tests which detect \( G_2 \) s-a-1
\[ G_2 = 0 \Rightarrow B = 1, \quad D = 1 \]
\[ G_1 = 0 \text{ (to propagate through } G_4) \Rightarrow A = 0 \text{ or } B = 0 \]
\[ G_3 = 0 \text{ (to propagate through } G_4) \Rightarrow C = 0 \text{ or } D = 0 \]

1. Test is \( \overline{A} \overline{B} \overline{C} \overline{D} = T_\beta \)

Tests which distinguish A s-a-0 from \( G_2 \) s-a-1
\[ = T_\alpha \oplus T_\beta = A \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C} \overline{D} = 1 \]

(b) The test \( A \overline{B} \overline{C} \overline{D} \) sensitizes both faults but the two errors cancel each other \((D + \overline{D}) = 1\) and does not detect the multiple fault.

\[ f_{\text{normal}} = A \overline{B} + C \overline{D} + \overline{B} \overline{D} = A \overline{B} + C \overline{D} + \overline{B} + \overline{D} = A + C + \overline{B} + \overline{D} \]

\[ f'_{x, 0} = (C \overline{B}) + C \overline{D} + 1 = 1 \text{ (multiple fault)} \]

\[ f \circ f'_{x, 0} = \overline{A} \overline{B} \overline{C} \overline{D} \text{ is only test which detects multiple fault but it does not sensitize } A \text{-s-a-0 fault} \]
Problem 2.6

(a) For fault A s-a-1
\[ \eta = 0 \left( B=1, C=0 \text{ or } B=0, C=1 \right) \]
\[ \text{to propagate} \quad \text{through } C_0 \quad \text{to propagate} \quad \text{through } C_3 \]
\[ T_a = \overline{A}BC + \overline{A}\overline{B}C \]
\[ (0,1,0) \text{ or } (0,0,1) \]

(b) For fault b s-a-o
\[ b=1 \rightarrow C=01, \overline{A}=1 \rightarrow A=0 \]
\[ T_b = \overline{A}C \rightarrow (0,0,1) \]

(c) If A=1 both A s-a-1 and b s-a-o are detected.
Therefore, test for multiple fault requires A=0.
If C=0 b s-a-o fault is desensitized and if B=1
A s-a-1 fault is propagated to the output.
Therefore, test for b s-a-o detects the multiple fault.
If C=1 the fault A s-a-1 propagates onto b s-a-o line
resulting in error signal D on b. If B=1 A s-a-1
fault also propagates to or gate resulting in D+D=1 at
output (error cancel). If B=0 fault on s propagates
to output. Therefore test for A s-a-1 also detects
multiple fault.

Check formal = AB + AC
\[ f_{s,a,b} (\text{mult. fault}) = 1, B + C = B + C = B \]
\[ f_{s,a,b} = \overline{A}BC + \overline{A}\overline{B}C \quad \text{detected} \]
multiple fault A s-a-1, b s-a-o.

(d) \[ T_a \oplus T_b = (\overline{A}BC + \overline{A}\overline{B}C) \oplus \overline{AC} \]
\[ = \overline{A}RE + \overline{A}\overline{B}C \]
\[ = (0,0,1) \quad (0,1,0) \]
\[ (0,0,-) \quad \text{distinguishes } 2 \text{ faults} \]
\[ f_c \oplus f_d = \overline{A}C \]
\[ f_c \oplus f_d = \overline{A}C \]