Solutions to Quiz 1 — CSE 105, Winter ’04

Problem 1 [6 points]

Let $L$ be the language of all strings $\omega \in \{0, 1\}^*$, in which every substring 010 is immediately followed by substring 111. (E.g. 101011101101110 and 11011011 are in $L$, but 10100111 or 0010111011011110 are not.) Draw the state diagram of a DFA that accepts $L$.

![DFA Diagram]

Figure 1: DFA

Problem 2 [4 points]

Give a regular expression for the language $L$ of all strings $w \in \{a, b\}^*$ such that the second character is an “a” and the last two characters are the same. E.g., “babbbab” and “aaa” are in the language, but “bbaa” or “aaba” are not.

$$(a \cup b)a(a \cup b)^*(aa \cup bb) \cup aaaS \cup baa \cup aa$$

Problem 3 [8 points]

Transform the following NFA into an equivalent DFA using the procedure studied in class. (Do not draw the non-reachable states.)
Figure 2: NFA

Figure 3: DFA
Problem 4 [12 points]

Indicate whether the following statements are TRUE or FALSE by circling correct answers. Briefly justify your answers.

(a) $\emptyset^* = \emptyset$

FALSE: $\emptyset^* = \{\varepsilon\}$, $\emptyset = \{\}$

(b) If every string in a language $L$ is accepted by some DFA, then $L$ is regular.

FALSE: Take any non-regular language $L$. Every string in $L$ is accepted by a DFA accepting $\Sigma^*$.

(c) If $M = (Q, \Sigma, \delta, q_0, F)$ is a DFA and $F = Q$, then $L(M) = \Sigma^*$.

TRUE: For every $\omega \in \Sigma^*$ the path from $q_0$ defined by $\omega$ ends in an accepting state.

(d) If $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA and $F = Q$, then $L(M) = \Sigma^*$.

FALSE: Consider an NFA where all arrows are $\varepsilon$ transitions.

(e) If $L$ is accepted by an $n$-state NFA then $L$ is accepted by some $3^n$-state DFA.

TRUE. We studied that any $n$-state NFA can be converted into an equivalent $2^n$-state DFA. Obviously, useless states can be added.

(f) If $L$ is a non-regular language and $F$ is a finite language, then $L \cap F$ is a regular language.

TRUE. $L \cap F$ is a finite language, and every finite language is regular.

Problem 5 [10 points]

Using the pumping lemma prove that $L = \{0^i1\omega : i \geq 0, \omega \in \{0, 1\}^*, |\omega| \leq i\}$ is not regular.

Assume that $L$ is regular. Let $p$ be the pumping length. Let $\omega = 0^p11^p$. Note that $\omega \in L$ and $|\omega| \geq p$. According to the pumping lemma we can write $\omega = xyz$, where $|xy| \leq p$ and $|y| > 0$. So $x = 0^a, y = 0^b, z = 0^{p-a-b}11^p$ and $b > 0$ and $a + b \leq p$. Pumping down ($i = 0$) we get $xz = 0^{p-b}11^p$, where $p > p - b$. So $xz \notin L$, which contradicts the Pumping lemma. Hence $L$ is not regular.

Problem 6 [10 points]

A string $x$ is a suffix of a string $y$ if there exists some string $z$ such that $y = zx$. (E.g., $abab$ is a suffix of $bbaabab$ or $abab$, but not of $ababbbab$.) Let $L$ be a language over an alphabet $\Sigma$. Define $\text{Suffix}(L) = \{x : x$ is a suffix of some $y \in L\}$. Note that $L \in \text{Suffix}(L)$. Prove
that the class of a regular language is closed under the suffix operation, i.e., if \( L \) is regular, then \( \text{Suffix}(L) \) is also regular.

Proof: Assume \( L \) is regular and let \( M = (Q, \Sigma, \delta, s, F) \) be a DFA such that \( L = L(M) \). We prove that \( \text{Suffix}(L) \) is regular by giving an NFA \( N = (Q', \Sigma, \delta', s', F') \) such that \( L(N) = \text{Suffix}(L) \). Let \( T \) be the set of states \( q \in Q \) that can be reached from the start state in \( M \):

\[
T = \{ q \in Q : \exists r_0, \ldots, r_n \in Q, \exists a_1, \ldots, a_n \in \Sigma. r_0 = s, r_i = \delta(r_{i-1}, a_i), q = r_n \}
\]

The idea is to make all states in \( T \) possible initial states. Since DFAs and NFAs can have only one start state, we define \( N \) by introducing a new start state \( s' \) and adding \( \epsilon \) transitions from \( s' \) to all states in \( T \). The formal definition of \( N \) is:

- \( Q' = Q \cup \{s'\} \)
- \( s' \) is a new state
- \( F' = F \).
- \( \delta'(s', \epsilon) = T, \delta'(q, a) = \{\delta(q, a)\} \) for all \( q \in Q \) and \( a \in \Sigma \), and \( \delta'(q, x) = \emptyset \) for all other cases.

The resulting NFA accepts \( \text{Suffix}(L) \). Hence \( \text{Suffix}(L) \) is regular.