Mosaics, Plenoptic Function, and Light Field Rendering

Topics in Image-Based Modeling and Rendering
CSE291 J00
Lecture 3

Last Lecture

- Camera Models
  - Pinhole perspective
  - Affine/Orthographic models
- Homogeneous coordinates
- Coordinate transforms
- Lenses
- Radiometry
  - Irradiance
  - Radiance
  - BRDF
Pinhole cameras

- Abstract camera model - box with a small hole in it
- Pinhole cameras work in practice

The equation of projection:
Mapping from 3-D world coordinates to 2-D image coordinates

Cartesian coordinates:
- We have, by similar triangles, that 
  \((x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z}, -f)\)
- Ignore the third coordinate, and get

\((x, y, z) \rightarrow (\frac{f x}{z}, \frac{f y}{z})\)
The camera matrix

Turn previous expression into Homogenous Coordinates
– HC’s for 3D point are (X,Y,Z,T)
– HC’s for point in image are (U,V,W)

Affine Camera Model

• Take Perspective projection equation, and perform Taylor Series Expansion about (some point \((x_0, y_0, z_0)\).
• Drop terms of higher order than linear.
• Resulting expression is affine camera model
Orthographic projection

Take Taylor series about \((0, 0, z_0)\) – a point on optical axis

Coordinate Changes: Rigid Transformations

\[
^B P = ^B R ^A P + ^B O_A
\]
Block Matrix Multiplication

\[
A = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \quad B = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
\]

What is \( AB \)?

\[
AB = \begin{bmatrix}
A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\
A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22}
\end{bmatrix}
\]

Homogeneous Representation of Rigid Transformations

\[
\begin{bmatrix}
B P \\
1
\end{bmatrix} = \begin{bmatrix}
A R & B O_A \\
0^T & 1
\end{bmatrix}
\begin{bmatrix}
A P \\
1
\end{bmatrix} = \begin{bmatrix}
A R A P + B O_A \\
1
\end{bmatrix} = \begin{bmatrix}
B^T \\
1
\end{bmatrix}
\begin{bmatrix}
A P \\
1
\end{bmatrix}
\]

Camera parameters

- **Issue**
  - camera may not be at the origin, looking down the z-axis
  - extrinsic parameters
    - one unit in camera coordinates may not be the same as one unit in world coordinates
  - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

\[
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix} = \begin{bmatrix}
\text{Transformation representing intrinsic parameters} \\
\text{Transformation representing extrinsic parameters}
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
T
\end{bmatrix}
\]

\(3 \times 3\) \(4 \times 4\)
The reason for lenses

Thin Lens: Image of Point

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]
Radiometry

- Solid Angle
- Irradiance
- Radiance
- BRDF
- Lambertian/Phong BRDF

Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point

- The solid angle subtended by a patch area \( dA \) is given by

\[
d\omega = \frac{dA \cos \theta}{r^2}
\]
Radiance

- Power is energy per unit time

- **Radiance**: Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle

- Symbol: \( L(x, \theta, \phi) \)

- Units: watts per square meter per steradian: \( \text{w/(m}^2 \text{sr}^1) \)

\[
L = \frac{P}{(dA \cos \theta) d\omega}
\]

Irradiance

- How much light is arriving at a surface? \( E(x) \)

- Units \( \text{w/m}^2 \)

- Incident power per unit area *not foreshortened*

- This is a function of incoming angle.

- A surface experiencing radiance \( L(x, \theta, \phi) \) coming in from solid angle \( d\omega \) experiences irradiance:

\[
L(x, \theta, \phi) \cos \phi d\omega
\]

- Crucial property: Total power arriving at the surface is given by adding irradiance over all incoming angles. Total power is

\[
\int_{\Omega} L(x, \theta, \phi) \cos \phi \sin \theta d\theta d\phi
\]
Radiance transfer

What is the power received by a small area $dA_2$ at distance $r$ from a small emitting area $dA_1$?

From definition of radiance

$$ L = \frac{P}{(dA \cos \theta) d\omega} $$

From definition of solid angle

$$ d\omega = \frac{dA \cos \theta}{r^2} $$

$$ P = \frac{L}{r^2} dA_1 dA_2 \cos \theta_1 \cos \theta_2 $$

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BRDF

With assumptions in previous slide

- Bi-directional Reflectance Distribution Function
  $$ \rho(\theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) $$

- Ratio of incident irradiance to emitted radiance

- Function of
  - Incoming light direction:
    $$ \theta_{\text{in}} \ , \ \phi_{\text{in}} $$
  - Outgoing light direction:
    $$ \theta_{\text{out}} \ , \ \phi_{\text{out}} $$

$$ \rho(x; \theta_{\text{in}}, \phi_{\text{in}}; \theta_{\text{out}}, \phi_{\text{out}}) = \frac{L_0(x; \theta_{\text{out}}, \phi_{\text{out}})}{L(x; \theta_{\text{in}}, \phi_{\text{in}}) \cos \phi_{\text{in}} d\omega} $$
Surface Reflectance Models

Common Models

• Lambertian
• Phong
• Physics-based
  – Specular
    [Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
  – Diffuse
    [Hanrahan, Kreuger 1993]
  – Generalized Lambertian
    [Oren, Nayar 1993]
  – Thoroughly Pitted Surfaces
    [Koenderink et al 1999]
• Phenomenological
  [Koenderink, Van Doorn 1996]

Arbitrary Reflectance

• Non-parametric model
• Anisotropic
• Non-uniform over surface
• BRDF Measurement
  [Dana et al. 1999], [Marschner 1]
Announcements

• Mailing list: cse291-j@cs.ucsd.edu
  Has been setup with class list as of Sunday night.
  If you’re not on it, and want to be added (e.g. auditing, and not on course list, send the email msg to majordomo@cs.ucsd.edu with body saying:
  subscribe cse291-j my_email@something.something

• Class presentations: requests from
  – Sameer Agarwal
  – Jin-Su Kim
  – Satya Mallick
  – Peter Schwer
  – Diem Vu
  – Cindy Wang
  – Yang Yu

This lecture

S. Chen, Quicktime VR - an image-based approach to virtual environment navigation, SIGGRAPH, pages 29-38, Los Angeles, California, August 1995.


M. Levoy, P. Hanrahan  Light Field Rendering , SIGGRAPH, 1996
Traditional Modeling and Rendering

Modeling → Rendering

- User Input: texture maps, survey data
- Geometry, Reflectance, Light sources
- Images

For Photorealism:

Modeling is Hard  Rendering is Slow

Can we model and render this?
What do we want to do with the model?
Mosaics & Quicktime VR

View Synthesis Without Motion Analysis

(Peri and Nayar, 1997)
(Shum and Szeliski, 1998)
(Quicktime VR, Chen, 1995)

Constructing Mosaic: Cylindrical images

- Easy to acquire with camera & tripod.
- Any two planar perspective projections of a scene which share a common viewpoint are related by a two-dimensional projective transform.

\[
\begin{bmatrix}
  u_2 \\
  v_2 \\
  w_2
\end{bmatrix}
= 
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} \\
  a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33}
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  v_1 \\
  w_1
\end{bmatrix}
\]

- Can be estimated from a minimum of four points correspondences in two images.
Stitched panoramic image from photos

Figure 9. A stitched panoramic image and some of the photographs the image stitched from.

Perspective view created from region

Figure 5. A perspective view created from warping a region enclosed by the yellow box in the panoramic image.
QuickTime VR

- continuous camera panning and zooming, jumping to selected points.
- cylindrical environment maps or panoramic images to accomplish camera rotation.

![Figure 2](http://www.pbs.org/wgbh/nova/pyramid/explore/khufuenter.html)

Quicktime VR: Example

http://www.pbs.org/wgbh/nova/pyramid/explore/khufucenter.html
Plenoptic Function

Images as a Collection of Rays

An image is a subset of the rays seen from a given point. This "space" of rays occupies two dimensions.

From Leonard McMillan’s, SIGGRAPH 99 course notes.
Where to Begin?

✓ Pinhole camera model

- Defines a mapping from image points to rays in space

$P(x)$

Mapping from Rays to Points

✓ Simple Derivation

$$P = \begin{bmatrix} u_x & v_x & 0_x \\ u_y & v_y & 0_y \\ u_z & v_z & 0_z \end{bmatrix}$$

$$X = \hat{C} + t \hat{P} \hat{x}$$

From Leonard McMillan’s, SIGGRAPH 99 course notes

CS348, Fall 2001 © David Kriegman, 2001
The Plenoptic Function

- The set of rays seen from all points ...

\[ p = P(\theta, \phi, x, y, z, \lambda, t) \]

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Image-based rendering is about

- reconstructing a plenoptic function from a set of samples taken from it.

- Ignoring time, and selecting a discrete set of wavelengths gives a 5-D plenoptic function.
Lumigraph/Lightfield


M. Levoy, P. Hanrahan, Light Field Rendering, SIGGRAPH, 1996.

Historical roots in:

Radiance properties

• In free space, radiance is constant as it propagates along a ray
  – Derived from conservation of flux
  – Fundamental in Light Transport.

\[ d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2 \]
\[ d\omega_1 = dA_2 / r^2 \quad d\omega_2 = dA_1 / r^2 \]
\[ d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2 \]
\[ \therefore L_1 = L_2 \]
Radiance is constant along straight lines

- Power 1→2, leaving 1:
  \[ L(x_1, \theta, \phi)(dA_1 \cos \theta_1) \left( \frac{dA_2 \cos \theta_2}{r^2} \right) \]
- Power 1→2, arriving at 2:
  \[ L(x_2, \theta, \phi)(dA_2 \cos \theta_2) \left( \frac{dA_1 \cos \theta_1}{r^2} \right) \]
- But these must be the same, so that the two radiances are equal

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Quiz

Does the brightness that a wall appears to the eye depend on the distance of the viewer to the wall?
Light Field/Lumigraph Main Idea

- In free space, the 5-D plenoptic function can be reduced to a 4-D function (radiances) on the space of light rays.
- Camera images measure the radiance over a 2-D set – a 2-D subset of the 4-D light field.
- Rendered images are also a 2-D subset of the 4-D lumigraph.