BRDF’s and Relighting

Topics in Image-Based Modeling and Rendering
CSE291 J00
Lecture 12

Environment Matte

Basic Assumption: Single ray into object, single ray out.
Background textures w/ foreground object

Background Textures

Note: Complex O(log k)

Explanation of $\Phi$

$$\Phi = \sum_{i=1}^{m} R_i M(T_i, A_i)$$

$R_i$: Reflectance coefficient
$M$: Texture mapping operator for axis-aligned rectangle ($A_i$) of texture ($T_i$)
Environment Matte Example

Alpha Matte  Environment Matte  Photograph

A composited object
BRDF

Surface Reflectance Models

Common Models

- Lambertian
- Phong
- Physics-based
  - Specular
    - [Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
  - Diffuse
    - [Hanrahan, Kreuger 1993]
  - Generalized Lambertian
    - [Oren, Nayar 1995]
  - Thoroughly Pitted Surfaces
    - [Koenderink et al 1999]
- Phenomenological
  - [Koenderink, Van Doorn 1996]

Arbitrary Reflectance

- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement
  - [Dana et al, 1999]
BRDF

Function of Three variables

Isotropic BRDF

$f_r(p, \theta_i, \phi_i, \theta_e, \phi_e) = f_i(\theta_p, \phi_p, \theta_e, \phi_e)$
BSSRDF

Bidirectional Subsurface Scattering Reflectance Distribution Function

Off Specular Reflection

Adapted from Steve Marschner
Backscatter

BRDF

\[ f_r(p, \theta_i, \phi_i, \theta_e, \phi_e) \]
BSSRDF

Bidirectional Subsurface Scattering Reflectance Distribution Function

Materials: Conductor

Conductor

Conductor + Microgeometry
Material: Insulator

Measured BRDFs
Ward reflectance model

- A physically realizable variant of the Phong model (satisfies energy conservation and reciprocity).

\[ \rho_d: \text{ proportion of incident radiation reflected diffusely.} \]
\[ \rho_s: \text{ proportion of incident radiation reflected specularly.} \]
\[ \alpha: \text{ surface roughness, or blur in specular component.} \]

Cook-Torrance Model (1982)

- Diffuse (Lambertian) and Specular and Fresnel reflection
- Microfacet model – surface is modeled as a collection of parallel symmetric V-groves called microfacets (facets are large w.r.t. wavelength, small w.r.t. pixel size).

- Facet distribution is given by a specific distribution (e.g., Gaussian, \( D = k \exp(\alpha/m)^2 \))
- Facets are purely specular
Geometric Attenuation: Masking and Shadowing

Geometric Attenuation

\[ G = 1-\frac{L_1}{L_2} \]

Fresnel Equation for Polished Copper
Reflectance as Function of Angle of Incidence for Copper

Blue  Green  Red

Generalized Lambertian Model (Oren, Nayar 1994)

- Like Torrance-Sparrow, but with Lambertian facets.
- Intensity doesn’t fall off as quickly as function of incident illumination.
Velvet: A general BRDF

Portrait of Sir Thomas More, Hans Holbein the Younger, 1527

[ After Koenderink et al., 1998 ]

Measuring Isotropic BRDF’s

Adapted from Steve Marschner
Image-based: Marschner

- Known Geometry of Sample,
- From single image, one obtains 2-D slice of BRDF.

Known illumination: inverse rendering

- If one assumes that illumination and surface geometry are known in advance, one can recover samples of the BRDF from an image (Marschner; Sato & Ikeuchi).
Empirical BRDF’s

Consider a collection of basis functions $b_j(\theta_i, \phi_i, \theta_r, \phi_r)$

Represent an arbitrary BRDF as

$$\rho(\theta_i, \phi_i, \theta_r, \phi_r) = \sum_j w_j b_j(\theta_i, \phi_i, \theta_r, \phi_r)$$

Given measurements, estimate $w_j$ to fit BRDF to data.

What is a good set of basis functions?

1. Product of spherical harmonics
2. Wavelets
3. Zernike polynomials

Spherical Harmonics

A set of orthonormal basis functions defined on the unit sphere.

Definition of SPHERICAL HARMONICS:

The first nine harmonics: $Y_{lm}(\theta, \phi)$

-Basri, Jacobs'01; Ramamoorthi, Hanrahan'01

(Borrowed from: Ramamoorthi, Hanrahan, SIGGRAPH'01)

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Phenomenological BRDF model
Zernike Polynomials
[Koenderink & van Doorn, 1996]

- A problem with spherical harmonics, half of sphere should be zero.
- General compact representation defined on disk
- Preserve Helmholtz Reciprocity
- Preserve reciprocity/isotropy if desired
- Domain is product of hemispheres
- Same topology as unit disk, adapt basis

Zernike Polynomials

- Optics, complete orthogonal basis on unit disk using polynomials of radius

\[ Z_n^m(\rho, \phi) = \frac{\sqrt{n+1}}{\pi} R_n^m(\rho) e^{im\phi} \]

- \( R \) has terms of degree at least \( m \). Even or odd depending on \( m \) even or odd
- Orthonormal, using measure \( \rho d\rho d\phi \)

Cool Demo: [http://wyant.opt-sci.arizona.edu/zernikes/zernikes.htm](http://wyant.opt-sci.arizona.edu/zernikes/zernikes.htm)
“Relighting”

1. Steerable lighting
2. Lambertian Surfaces and linear subspaces
3. Arbitrary BRDF, arbitrary lighting
Superposition of lighting: An important point

If $I_1 = R(S, L_1)$ is the image of a scene $S$ under lighting $L_1$ and if $I_2 = R(S, L_2)$ is the image of a scene $S$ under lighting $L_2$,

then the image of the scene under lighting $L_1 + L_2$ is simply

$$I_1 + I_2$$

Basis functions: Example

Consider sinusoid of frequency $f$, we can specify any sinusoid as sum of two basis sinusoids $\cos(ft)$ and $\sin(ft)$ as:

$$a \cos(ft) + (1-a) \sin(ft)$$

Such basis function are sometimes called steerable functions – over some transformation of the parameter (e.g. $t$ here, $(x,y)$ for image plane), the function can be represented as the linear combination of a finite collection of basis functions.
Steerable lighting for relighting

1. Choose a steerable basis for the lighting – all rendering lighting conditions will be defined as linear combinations of the basis lighting.
2. Gather (or synthesize) images of a scene under the basis lighting.
3. Render new images by taking linear combinations of basis images.

Why does it work? Superposition

Lambertian Surface: $\rho(\theta_{\text{in}}, \phi_{\text{in}} ; \theta_{\text{out}}, \phi_{\text{out}}) = \text{constant}$

At image location $(u,v)$, the intensity of a pixel $I(u,v)$ is:

$$I(u,v) = [a(u,v)\hat{n}(u,v)] \cdot [s_0\hat{s}] = b(u,v) \cdot s$$

where

- $a(u,v)$ is the albedo of the surface projecting to $(u,v)$.
- $\hat{n}(u,v)$ is the direction of the surface normal.
- $s_0$ is the light source intensity.
- $\hat{s}$ is the direction to the light source.
**Image Formation Model: No shadows**

Lambertian model without shadowing:

\[ I = B \cdot s \]

where
- \( I \) is an \( n \)-pixel image vector
- \( B \) is a matrix whose rows are unit normals scaled by the albedos
- \( s \in \mathbb{R}^3 \) is a vector of the light source direction scaled by intensity

**Image Formation Model: Convex Object**

Lambertian model with attached shadows:

\[ I = \max(B \cdot s, 0) \]

where
- \( I \) is an \( n \)-pixel image vector
- \( B \) is a matrix whose rows are unit normals scaled by the albedos
- \( s \in \mathbb{R}^3 \) is a vector of the light source direction scaled by intensity
**Illumination Subspace**

\[ L = \{ I | I = Bs, \text{ for all } s \in \mathbb{R}^3 \} \]

- \( L \) is a 3-D linear subspace of image space, \( \mathbb{R}^n \).
- \( L \) is spanned by 3 linearly independent images.
- Cone can be generated from \( L \).
- See also [Woodham 81], [Shashua 92], [Hallinan 95], [Hayakawa 94], [Rosenholtz, Koenderink 96]

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**Multiple Sources: No shadows**

- Consider two sources.
- Light source 1: \( I_1 = Bs_1 \)
- Light source 2: \( I_2 = Bs_2 \)
- Image with both lights on: \[ I = I_1 + I_2 = Bs_1 + Bs_2 = B(s_1 + s_2) \]

So, what does this mean?
Computing \( \mathbf{L} \)

For \( k \) images \( \mathbf{X} = [ \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k ] \) imaged under \( k \) unknown point light sources \( \mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_k] \).

\( \mathbf{X} = \mathbf{B} \mathbf{S} \)

Given \( k \geq 3 \) images we can compute \( \mathbf{B}^* \) that spans \( \mathbf{L} \) with

1. singular value decomposition
2. or methods robust to outliers.

Still Life

Original Images

Basis Images
Samples from the Cone

Face Basis

Original Images

Basis Images spanning L
Image-Based Rendering: Attached Shadows

Single Light Source Face Movie

Lumigraph/Light Field Rendering

- Set of light rays is a 4-D manifold.
- Single camera image provides a 2-D sampling of the real scene’s radiance.
- Moving the camera over a 2-D surface yields a sampling of 4-D radiance (light) field \( L(u,v,s,t) \).
- Assume lighting is fixed.
Lumigraph/Light Field Rendering

Image from new viewpoint but under same lighting is rendered by indexing each visual ray into the Lumigraph

\[ I(x,y) = L(u,v,s,t) \]

where \( u,v,s,t \) are functions of \( (x,y) \).

A not so good idea

- Scene depth is needed.
- Depth reconstruction method.
- Rendering method.

Can we synthesize images under novel lighting by indexing into the image set?
Why is depth needed? Correspondence

- Isotropic point light source in known position.
- Calibrated camera

- Which pixel corresponds to which light source ray?
- Which light source ray indirectly illuminates which pixel?

Depth resolves this.
Scene Depth

We would like to recover the depth $\lambda$:

$$p(\kappa) = o + \lambda \vec{r}$$

Reconstruction from the Illumination Field

1. Consider fixed camera and point light source
2. Light moves over a star-shaped surface

Intensity $I_1(\phi, \psi)$ as function of $(\theta, \psi)$
Intensity of One Pixel: \( I_1(\phi_1, \psi_1) \)

This is effectively a 2-D slice of a surface point’s BRDF except for
- Shadowing
- Variation in the distance between the sources and the surface point.

Double Covering of the Illumination Field

Consider the effect of moving the light over a second surface:

Intensity \( I_2(\phi, \psi) \) as a function of \((\phi, \psi)\)
\( I_1(\phi_1, \psi_1) \) and \( I_2(\phi_2, \psi_2) \)

**Inner Sphere:**
\[ I_1(\phi_1, \psi_1) \]

**Outer Sphere:**
\[ I_2(\phi_2, \psi_2) \]

**Image Acquisition**
Relation Between Intensity Maps

When the surface point $p, s_1(\phi_1, \psi_1)$ and $s_2(\phi_2, \psi_2)$ are collinear (in correspondence), the measured pixel intensities are simply related by the relative $1/r^2$ losses.

Depth Estimation

This correspondence can be expressed as a change of coordinates $\phi_2(\phi_1, \psi_1; \lambda)$ and $\psi_2(\phi_1, \psi_1; \lambda)$ parameterized by the depth $\lambda$. We can then estimate $\lambda$ by minimizing

$$O(\lambda) = \int \int \left[ I_2(\phi_2, \psi_1; \lambda), \psi_2(\phi_1, \psi_1; \lambda) \right] - d^2(p(\lambda)) I_1(\phi_1, \psi_1) d\phi_1 d\psi_1$$

where

$$d^2(p(\lambda)) = \frac{\|p(\lambda) - s_1\|^2}{\|p(\lambda) - s_2\|^2}$$
An apple and its depth map

A Reconstructed Depth Map

143 Images on each surface
Rendering Synthetic Images

Point Source Example

- For a given image point, there is a scene point: \( P \)
- Intersect light ray through \( P \) with sphere.
- Find triangle of light sources containing \( P \).
- Interpolate pixel intensities of images corresponding to the triangle vertices & scale by \( 1/r^2 \).

Rendered Images

![Rendered Images]
Indexing and Interpolation: Pixel by Pixel

Application: Rendering Isolated Objects

- Close Point Source
- Far Point Source
- Area Source
- Line Source
Application: Rendering Isolated Objects

Point Source (not on captured surface)          Multiple Sources - Area Source

Rendered Image: A Sea Shell

Isotropic point light source located between acquisition spheres.
Moving Light source

Moving Light source
A Moving Light Source

A comparison

Real Image  Rendered Image

Magnitude of difference Image

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Embedding Objects in Synthetic Scenes

- Blue Moon Rendering Tools to render scene
- Custom surface shader to implement rendering method by indexing the illumination dataset