Problem 1.

Since the language \{ww^r | w \in \{0,1\}^*\} is context free, we can construct a PDA recognizing the language \{ww^r | w \in \{0,1\}^*\}. Next we combine this PDA with an input DFA to another PDA M’. If the set of languages recognized by M’ is not empty, then it means the input DFA can accept some string in the form of ww^r, Otherwise, the input DFA does not accept any string in the form of ww^r. That is to say, the input DFA belongs to L1 if and only if L(M’) is not empty. Since the emptiness testing problem for the language recognized by a PDA is decidable, then L1 is decidable.

Problem 2.

Given a string <M, w>, let n be the number of symbols in w, q the number of states of M, t the size of alphabet. The key point of this problem is if M’s head is restricted in the first n cells, then in \(qnt^n + 1\) steps M must halt or loop. At that time we can say

\(<M, w> \in L_2.\)

We construct following M’ deciding L2.

M’= “On input <M, w>

1. Run M on w for \(qnt^n + 1\) steps

2. If the head of M is beyond the nth tape cell, reject;

3. If M halts, accept;

4. Accept.”

Problem 3.

We use reduction from HALT_{TM} to \(\overline{L3}\) to prove the undecidability of L3. (Decidability is not affected by complementation.)

We construct a function reducing HALT_{TM} to \(\overline{L3}\).
\(<M', w'> = f(M, w)\) in which if \(<M, w> \in \text{HALT}_{TM}\), then \(<M', w'> \in \overline{L3}\).

otherwise \(<M', w'>\) doesn’t belong to \(\overline{L3}\).

Let \(w' = '\#'+w\).

Following F computes a function \(f\):

\(F= "\text{On input } <M, w>\)

1. Construct TM \(M'\)

\(M' = "\text{On input x}\)

1. Move the head of \(M'\) right thus skip the first symbol, the rest of input is called \(x'\)
2. Run \(M\) on \(x'\)
3. If \(M\) halts, then modify the first tape cell of \(M'\), accept.

2. Output \(<M', w'>\)

Now we can say \(<M', w'> \in L_3\), if and only if \(<M, w> \in \text{HALT}_{TM}\). Since \(\text{HALT}_{TM} \leq_m \overline{L3}\) and \(\text{HALT}_{TM}\) is undecidable, then \(\overline{L3}\) is undecidable, \(L_3\) is undecidable.