Problem 1.

The PDA recognizes the language by guessing the length of the input at the beginning. For the guess that the length is even it follows the upper branch, while for the guess that length is odd it follows the lower branch.

(1) In the upper branch, the PDA accepts even length strings by pushing and popping alternatively, starting with the second symbol read and ending before the last symbol.

(2) In the lower branch, the PDA first pushes the symbols onto stack and guesses the middle one then pops out the symbols one by one while reading symbols after the guessed middle symbol. If all the symbols except the first one and the start symbol “$” can be popped out when the PDA finishes reading the symbol except the last one, then the guess is right. The lower branch remembers the first symbol by developing two branches. Each branch accepts when the guessed middle symbol is different from the first one.
(3) Before entering accept state, the PDA only pops out the first read symbol out of the stack when the last one read is the same. Thus it only accepts the string starting and ending with the same symbol.

**Problem 2**

\[ L' = (a^i b^j c^k) \cup (\Sigma^* (a^i b^j c^k)) \ (i \neq j \text{ or } j \neq k \text{ or } k \neq \epsilon) \]

**CFG for \( L'\):**

\[ S \rightarrow AC' | BC' | A'D | A'E | T \]

\[ A' \rightarrow aA' | \epsilon \]

\[ B' \rightarrow bB' | \epsilon \]

\[ C' \rightarrow cC' | \epsilon \]

\[ A \rightarrow aAb | aA' \]

\[ B \rightarrow aBb | B'b \]

\[ D \rightarrow bDc | cC' \]

\[ E \rightarrow bEc | bB' \]

\[ T \rightarrow XbaX | XcbX | XcaX \]

\[ X \rightarrow aX | bX | cX | \epsilon \]
Problem 3
Problem 4

$\varepsilon, \varepsilon \rightarrow a$

$\varepsilon, a \rightarrow \varepsilon$

$\varepsilon, e \rightarrow \varepsilon$

$\varepsilon, e \rightarrow b$

$\varepsilon, a \rightarrow e$

$\varepsilon, e \rightarrow b$

$\varepsilon, b \rightarrow e$

$\varepsilon, a \rightarrow e$

$\varepsilon, b \rightarrow e$

$\varepsilon, \varepsilon \rightarrow a$

$\varepsilon, a \rightarrow \varepsilon$

$\varepsilon, e \rightarrow a$

$\varepsilon, a \rightarrow e$

$\varepsilon, e \rightarrow b$

$\varepsilon, b \rightarrow e$

$e_{14} \rightarrow \Lambda_{23}$

$e_{16} \rightarrow \Lambda_{26}$

$e_{15} \rightarrow \Lambda_{24} b$
A24---a A33

A26---a A36

A25---a A34 b

A54--- A23

A56---A26

A55---A24 b
A44---->a A44 b

A46---->a A45

A46---->a A46

A44----> A54 b

A46----> A55

A46----> A56

A07----> A16

A00----> e, A22---->e, A33---->e, A44---->e, A55---->e, A66---->e, A77---->e
Aij->Aik Akj for all i, j, k