Problem 1

Draw the state diagram corresponding to the DFA $M = (Q, \Sigma, \delta, q_0, F)$ where $Q = \{A, B, C, D, E\}$, $\Sigma = \{0, 1\}$, $q_0 = A$, $F = \{B, C, D, E\}$ and $\delta$ is specified by the following table

$$
\begin{array}{|c|c|c|c|c|}
\hline
\delta & A & B & C & D & E \\
\hline
0 & A & C & E & B & D \\
1 & B & D & A & C & E \\
\hline
\end{array}
$$

Then, for each of the following input strings, give the corresponding computation of automaton $M$, and say whether the computation is accepting or rejecting. (Remember, the computation of a DFA on input $w$ is the sequence of states the automaton goes through when reading $w$.)

1. 1001
2. 0111
3. 1110

Optional: Can you give an English description of the strings accepted by the automaton?

Problem 2

For each of the following languages give a corresponding deterministic finite state automaton (DFA). The automaton should be specified by giving the associated state transition diagram.

1. The set $L_1$ of all strings over $w \in \Sigma = \{a, b\}$ such that $w$ does not contain $aaa$ or $bb$ as a substring. E.g., $ababaab, baabab \in L_1$, but $abbbaa, babaaabaab \notin L_1$.

2. The set $L_2$ of all strings over $w \in \Sigma = \{a, b\}$ such that $w$ contains at least two (not necessarily consecutive) $a$'s and at least two (not necessarily consecutive) $b$'s. E.g., $ababa, bbababab \in L_2$, but $bbbbbbaaab, aab \notin L_2$.

Problem 3

Transform the NFA from Figure ?? into an equivalent DFA using the transformation studied in class. (Notice: you should follow the transformation as described in the book, but you do not need to include unreachable states, i.e., states that can never be reached when starting from the initial state.)
Problem 4

Exactly two of the following five languages are regular. Identify them, and prove your answer correct. (You still do not know how to prove that a language is not regular. Here you only need to find the two regular languages, and prove their regularity. In order to prove that a language is regular you can either give a DFA or an NFA accepting the language.) All languages are defined over the alphabet $\Sigma = \{a, b\}$.

- $L_1 = \{w \in \Sigma^* : \text{the length of } w \text{ is even}\}$.
- $L_2 = \{w \in \Sigma^* : \text{the length of } w \text{ is a square number}\}$.
- $L_3 = \{w \in \Sigma^* : \text{the length of } w \text{ is odd and the middle character of } w \text{ is an } a\}$.
- $L_4 = \{w \in \Sigma^* : w \text{ contains more } a's \text{ than } b's\}$.
- $L_5 = \{w \in \Sigma^* : \text{the length of } w \text{ is at least 3, and the third from the last character of } w \text{ is a } b\}$.

Solution Problem 1

1001 $\rightarrow$ $A B C E E$, the computation is accepting
0111 $\rightarrow$ $A A B D C$, the computation is accepting
1110 $\rightarrow$ $A B D C E$, the computation is accepting
DFA $M$ recognizes all binary numbers which are not divisible by 5.

**Solution Problem 2**

![State Diagram for DFA of language $L_1$](image1)

Figure 3: State Diagram for DFA of language $L_1$

![State Diagram for DFA of language $L_2$](image2)

Figure 4: State Diagram for DFA of language $L_2$
Solution Problem 3

DFA transition function:

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\emptyset$</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
<th>$BC$</th>
<th>$AC$</th>
<th>$BCD$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$BC$</td>
<td>$BC$</td>
<td>$D$</td>
<td>$\emptyset$</td>
<td>$BCD$</td>
<td>$BCD$</td>
<td>$BCD$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>$AC$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$BC$</td>
<td>$\emptyset$</td>
<td>$AC$</td>
<td>$BC$</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 5: DFA State Diagram](image)

Solution Problem 4

The languages $L_1$ and $L_5$ are regular.

![Figure 6: State Diagram for DFA of language $L_1$](image)

![Figure 7: State Diagram for NFA of language $L_5$](image)