Due: beginning of class on Thu. March 14, 2002

Problem 1

Consider the language

\[ L_1 = \{ \langle M \rangle : M \text{ is a DFA and } w w^R \in \mathcal{L}(M) \text{ for some } w \in \{0, 1\}^* \} \]

i.e., the set of all strings \( s \) such that \( s \) is the encoding of a DFA that accept some binary string of the form \( w w^R \in \mathcal{L} \).

Prove that language \( L_1 \) is decidable.

Problem 2

Consider the language \( L_2 \) of all strings \( \langle M, w \rangle \) such that \( M \) is a Turing machine, \( w \) an input string, and on input \( w \), \( M \) only uses the part of the tape initially occupied by the input. (Informally, this language corresponds to the problem of determining, given a program \( M \) and the initial content \( w \) of the memory, if the program is ever going to use more memory that what initially allocated.)

Prove that language \( L_2 \) is decidable.

Problem 3

Consider the language \( L_3 \) of all strings \( \langle M, w \rangle \) such that \( M \) is a Turing machine, \( w \) an input string, and on input \( w \), \( M \) never changes the content of the first cell on the tape. (Informally, this language corresponds to the problem of determining, given a program \( M \) and the initial content \( w \) of the memory, if a specific memory cell correspond to a variable or a constant.)

Prove that language \( L_2 \) is undecidable.

Problem 4 (Optional for extra credit)

In problem 2, we considered the task of determining, given a program, and its input, if the program is ever going to use more memory than initially allocated. Here we consider the same problem, but without specifying the initial content (and size) of the memory. Namely, we consider the language \( L_4 \) of all strings \( \langle M \rangle \) such that \( M \)
is a Turing machine and for every input string $w$, $M$ never uses more memory than what initially allocated. Formally, $L_4 = \{ \langle M \rangle : \forall w. \langle M, w \rangle \in L_2 \}$.

Is $L_4$ decidable? Is $L_4$ recognizable? Is $L_4$ co-recognizable?