Due: beginning of class on Thu. March 7, 2002

Problem 1

Prove that the language

\[ L = \{0^n10^{2n}10^{3n} : n \geq 0\} \]

is not context free using the pumping lemma for context free languages.

Problem 2

In this course we have studied several computational models: DFA, NFA, PDA, TM. The first step in the study of a computational model is to give a formal definition of the machine. In this problem you are asked to give such formal definition for a new kind of automaton. Consider a deterministic automaton with an unbounded queue, i.e., an infinite memory that can be accessed using a first-in first-out discipline. At every step of the computation, the automaton reads one symbol from one end of the queue, and insert 0,1 or more symbols at the other end. (If the queue is empty, then a special symbol is read.) The input is initially stored on the queue, and the machine accept or reject the input by entering one of two special states.

A deterministic finite automaton with queue (QFA) can be defined as a 6-tuple \((Q, \Sigma, \Gamma, \delta, q_s, q_a, q_r)\). Where \(q_s, q_a, q_r \in Q\) are the start, accept and reject states. Answer the following questions:

- Give the formal definition of automaton with queue. Your definition should read something like “An automaton with queue is a 6-tuple \((Q, \Sigma, \Gamma, \delta, q_s, q_a, q_r)\) where \(Q\) is a finite set of states, etc.”

- Give the formal definition of configuration. Define also the start configuration, accept configurations and reject configurations.

- Define the computation relation \(\Rightarrow\), i.e., a transition relation on configurations that defines how the internal state of the automaton and the content of the queue evolve over time.

- Define the language accepted by the automaton.
Problem 3

Prove that QFA are not more powerful than Turing Machines, i.e., show that for every QFA \( M = (Q, \Sigma, \Gamma, \delta, q_s, q_a, q_r) \) there exists an equivalent TM \( M' = (Q', \Sigma, \Gamma', \delta', q_s, q_a, q_r) \), i.e., a TM accepting the same language as the original automaton.

In this problem, you should first give an informal (but detailed) description of \( M' \), explaining why your translation is correct.

Then, you should give a formal definition of \( M' \) specifying the components \( Q', \delta' \) etc. in terms of \( Q, \delta \) etc. (If you have difficulties formalizing \( M' \), give the informal description for partial credit. If your informal description if sufficiently detailed, you can get almost full credit without formal description.)

Problem 4 (Optional for extra credit)

Prove that every Turing Machine can be transformed into an equivalent QFA, i.e., show that for every TM \( M' = (Q', \Sigma, \Gamma', \delta', q_s', q_a', q_r') \) there exists a QFA \( M = (Q, \Sigma, \Gamma, \delta, q_s, q_a, q_r) \) that accepts the same language.

As in problem 3, you should first give an informal description explaining your construction, and then possibly formalize it giving the exact definition of \( M \).