Problem 1

Give a push down automaton that recognizes the language \( L \) of all strings \( w \in \{0,1\}^* \) such that the first and last symbol of \( w \) are the same, and moreover, if the length of \( w \) is odd, then the middle symbol of \( w \) is different from the first and the last. For example, strings 00, 11, 010, 101, 010100010010, 0101010 are in \( L \), but \( \epsilon, 0, 1, 000, 111, 110, 00001, 1001001 \) are not.

You should draw the state transition diagram of the PDA, (no formal definition required), and explain in english why your automaton is correct.

Problem 2

Consider the language

\[
L_{nm} = \{a^n b^n c^n : n \geq 0\}
\]

and let

\[
L' = \Sigma^* \setminus L_{nm}
\]

be its complement (i.e., \( L' \) is the set of all string \( w \in \{a, b, c\}^* \) such that \( w \) is not of the form \( a^n b^n c^n \).) Notice that \( L' \) includes strings like \( bac \) or \( abcabc \) where all symbols appear the same number of times, but in the wrong order.

Next week we will prove that the language \( L_{nm} \) is not context free. In this problem you are asked to prove that \( L' \) is context-free by giving a context free grammar that generates \( L' \). (In particular, this shows that the class of context free languages is not closed under the complement operation.)

Notice: in this problem you are explicitly required to give a CFG. Proving that \( L' \) is context free by giving a PDA is not allowed.
Problem 3
Transform the following grammar into a corresponding push down automaton using the procedure studied in class.

\[
S \rightarrow aSbSa|T \\
T \rightarrow aTb|bTa|\epsilon
\]

Problem 4
Transform the following push down automaton into a context free grammar using the procedure studied in class.

The grammar obtained from the transformation studied in class can probably be simplified, eliminating useless rules, etc. However, in this problem you should not perform any simplification: all rules resulting from the transformation should be listed in your solution.