**CSE 105: Introduction to the Theory of Computation** Winter 2001

# Problem Set 5

Instructor: Daniele Micciancio Mar. 6, 2001

**Due: beginning of class on Tue. Mar. 13, 2001**

## Problem 1

Consider the language

$$L = \{\langle M \rangle \mid M \text{ is a DFA and all strings accepted by } M \text{ are palindromes}\}$$

(Remember, $w$ is a palindrome if $w$ reads the same forward and backward.) Prove that $L$ is a decidable language. [Hint: first determine if the set of all strings that are *not* palyndromes is context free.]

## Problem 2

In class we have seen that the languages $A_{DFA}$, $A_{REX}$, $E_{DFA}$, $E_{REX}$, $ALL_{DFA}$, $ALL_{REX}$, $EQ_{DFA}$, $EQ_{REX}$, $A_{CFG}$, $A_{PDA}$, $E_{CFG}$, $E_{PDA}$ are decidable, but the languages $EQ_{CFG}$, $ALL_{CFG}$, $EQ_{PDA}$, $ALL_{PDA}$ are not. For each of the following languages say whether the language is decidable or not, and prove your answer by reduction to or from any of the above languages.

1. The set of all strings $\langle G, R \rangle$ where $G$ is a context free grammar, $R$ is a regular expression and $G$ and $R$ are equivalent, i.e., they generate the same language.
2. The set of all strings $\langle G, n \rangle$ such that $G$ is a context free grammar, $n$ a positive integer, and all strings generated by $G$ have length $n$.
3. The set of all strings $\langle N_1, N_2 \rangle$ such that $N_1, N_2$ are nondeterministic finite state automata, and $L(N_1) \subseteq L(N_2)$.

## Problem 3

In class we proved that the language $A_{TM}$ is Turing-recognizable, but not Turing-decidable.

1. Prove that $E_{TM}$ is also undecidable (You can either give a direct proof by diagonalization, or prove your answer by reduction from $A_{TM}$)
2. Is $E_{TM}$ Turing-recognizable? Is it co-Turing-recognizable? Briefly justify your answers.
3. Prove that $A_{TM}$ is not map-reducible to $E_{TM}$.