

Problem Set 4

Due: beginning of class on Tue. Mar. 6, 2001

This homework assignment contains only two problems. However the problems are longer than usual, so you are advised to start working on it as soon as possible.

Problem 1

For **each** of the following languages, say whether the language is regular, context-free or decidable, and prove your answer. In particular,

- if the language is regular give a *regular expression* for L .
- If L is context free, but not regular, give a *context free grammar* for L , **and** prove that L is not regular using the *pumping lemma for regular languages*.
- Finally, if L is decidable but not context free, give a *Turing machine* for L (the transition function for this Turing machine should be described with state diagram) **and** prove that L is not context free using the *pumping lemma for context free languages*.

(a)

$$L_1 = \{a^n b^m c^{n+m} \mid n, m \geq 0\}$$

(b)

$$L_2 = \{a^n b^{2n} c^{3n} \mid n \geq 0\}$$

(c)

$$L_3 = \{a^n b^{2m} c^{3k} \mid n, m, k \geq 0\}$$

Problem 2

A Turing machine with doubly infinite tape is defined similarly to an ordinary Turing machine, except that the memory tape is infinite in both directions. Initially the memory tape contains the input string, and the read/write head is positioned on the first character of the input. The formal definition is identical to ordinary Turing machines except that the *yields* relation gives

$$qb\beta \Rightarrow q' \sqcup c\beta$$

instead of $qb\beta \Rightarrow q'c\beta$ when $\delta(q, b) = (q', c, L)$. Prove that this type of Turing machine recognizes exactly the same class of languages as ordinary Turing machines. In particular you should

- (a) show that any ordinary Turing machine can be transformed into an equivalent Turing machine with doubly infinite tape
- (b) show that any Turing machine with doubly infinite tape can be transformed into an equivalent ordinary Turing machine.

Here “equivalent” means that the two machines recognize the same language. For both parts you should first give an informal description of the Turing machine you intend to build, followed by the formal definition of the machine.