

Problem Set 3

Due: beginning of class on Tue. Feb. 20, 2001

Problem 1

(a) Give a context-free grammar over the alphabet $T = \{x, +, *, (,)\}$ corresponding to all valid polynomial expressions in the symbol x , i.e., expressions built using symbol x , binary operations $+$, $*$ and parenthesis $(,)$ and obeying the usual syntactical rules of arithmetic. E.g., $(x + x) * x + x * x + ((x))$, $(x + x + (x * x) * (x + (x + (x + x))))$ are valid expressions, but $((x))$, $x(+x)$, or $x * +(x)$ are not.

(b) Give a left-most derivation of the string $(x + x) * (x + x * (x + x))$ using your grammar.

(c) Convert your grammar into an equivalent Push Down Automaton. (You can use generalized PDA if you like. Remember, a generalized PDA is a PDA with transition function of the form $\delta : Q \times \Sigma_\epsilon \times \Gamma_\epsilon \rightarrow \wp(Q \times \Gamma^*)$ that can push arbitrary strings $\gamma \in \Gamma^*$ onto the stack at each transition.) Give both the formal definition and the state diagram of the PDA.

Problem 2

(a) Give a Push Down Automaton for the language over the alphabet $\{a, b\}$ consisting of all words w that contains an equal number of a 's and b 's (in any order.) E.g., $aaabbb$ and $abbaabab$ are in the language, but $aabbb$ and $bbaabbaabb$ are not. (Here it is enough to draw the transition diagram. No formal definition required.)

(b) Give an accepting computation of your automaton on input $abbaba$. (Remember, a computation is a sequence of $w_i \in \Sigma_\epsilon$, $r_i \in Q$ and $s_i \in \Gamma^*$ satisfying the conditions in definition 2.8 from the book.)

(c) Transform the PDA into an equivalent context-free grammar.

Problem 3

In class we saw that the intersection of context-free languages is not always context-free, i.e., context-free languages are not closed under intersection. Prove that the intersection of a context-free language and a regular language is context-free.

[*Hint:* given a DFA M and a PDA N , show how M and N can be combined into a single PDA P such that the language accepted by P is the intersection of the languages accepted by M and N . *Additional Hint:* use construction similar to the one in Theorem 1.12 in the book for the intersection of regular languages. You will need to augment the construction with a stack.]

Problem 4

In the previous homework assignment, you proved that the language $L_4 = \{a^n b^m c^k \mid \min(n, m) \leq k\}$ is context-free. In this problem we examine the language

$$L = \{a^n b^m c^k \mid \max(n, m) \leq k\}.$$

Despite the similarity between the two definitions, it turns out that L is not context-free. Prove that L is not a context-free language using the pumping lemma for context-free languages.