CSE 105: Introduction to the Theory of Computation Winter 2001

Problem Set 2

Instructor: Daniele Micciancio Jan. 25, 2001

Due: beginning of discussion session on Fri. Feb. 2, 2001

(If you cannot attend the discussion, you can turn in your homework Thu Feb 1 in class, or give it to some other student to turn it in on Friday.)

Problem 1

One of the following two languages is regular and the other is not. Say which is which and prove your answer, either giving a regular expression for the language or proving non-regularity using the pumping lemma.

- $L_1 = \{a^n b^m \mid n \text{ and } m \text{ have the same reminder when divided by 3}\}$.
- $L_1 = \{a^n b^m \mid n \text{ and } m \text{ have the same quotient when divided by 3}\}$.

Problem 2

In the last problem set we defined the even and odd part of a string ($even(w_1 w_2 w_3 \ldots) = w_2 w_4 w_6 \ldots$ and $odd(w_1 w_2 w_3 \ldots) = w_1 w_3 w_5 \ldots$) and proved that if L is a regular language, then also $even(L) = \{even(w) \mid w \in L\}$ and $odd(L) = \{odd(w) \mid w \in L\}$ are regular. (See problem set 1 for details.)

Prove that the converse is not necessarily true, i.e. show that there exists a language L such that both $even(L)$ and $odd(L)$ are regular, but L is not regular. [Hint: one of the languages from problem 1.23 in the textbook should do.] Notice: after you choose a language L, you have both to prove that $odd(L)$ and $even(L)$ are regular (e.g., giving a regular expression or finite state automaton for them), and prove that L is not regular (using the pumping lemma and/or the closure properties of regular languages.)

Problem 3

For each of the following languages give a Push Down Automaton that accepts exactly that language.

- $L_3 = \{a^n b^m c^k \mid n + m \leq k\}$
- $L_4 = \{a^n b^m c^k \mid min(n, m) \leq k\}$

(You should describe the PDA's giving a state transition diagram. You are not required to give the formal description with the table for the transition function.)
[Optional (don't write anything, just think about this part): what can you say about the language $L_5 = \{a^n, b^m c^k \mid max(n, m) \leq k\}$? Can you give a Push down automaton for it?]

Problem 4

Give context free grammars that generates the languages L_3, L_4 from problem 3.
[Optional (don't write anything, just think about this part): as in problem 3, do you think the language $L_5 = \{a^n, b^m c^k \mid max(n, m) \leq k\}$ can be described by a context free grammar?]