Due: beginning of discussion session on Fri. Feb. 2, 2001

(If you cannot attend the discussion, you can turn in your homework Thu Feb 1 in class, or give it to some other student to turn it in on Friday.)

Problem 1

One of the following two languages is regular and the other is not. Say which is which and prove your answer, either giving a regular expression for the language or proving non-regularity using the pumping lemma.

- \( L_1 = \{a^n b^m \mid n \text{ and } m \text{ have the same reminder when divided by } 3\} \)
- \( L_2 = \{a^n b^m \mid n \text{ and } m \text{ have the same quotient when divided by } 3\} \)

Problem 2

In the last problem set we defined the even and odd part of a string \((\text{even}(w_1 w_2 w_3 \ldots) = w_2 w_4 w_6 \ldots \text{ and } \text{odd}(w_1 w_2 w_3 \ldots) = w_1 w_3 w_5 \ldots)\) and proved that if \(L\) is a regular language, then also \(\text{even}(L) = \{\text{even}(w) \mid w \in L\}\) and \(\text{odd}(L) = \{\text{odd}(w) \mid w \in L\}\) are regular. (See problem set 1 for details.)

Prove that the converse is not necessarily true, i.e. show that there exists a language \(L\) such that both \(\text{even}(L)\) and \(\text{odd}(L)\) are regular, but \(L\) is not regular. [Hint: one of the languages from problem 1.23 in the textbook should do.] Notice: after you choose a language \(L\), you have both to prove that \(\text{odd}(L)\) and \(\text{even}(L)\) are regular (e.g., giving a regular expression or finite state automaton for them), and prove that \(L\) is not regular (using the pumping lemma and/or the closure properties of regular languages.)

Problem 3

For each of the following languages give a Push Down Automaton that accepts exactly that language.

- \( L_3 = \{a^n b^m c^k \mid n + m \leq k\} \)
- \( L_4 = \{a^n b^m c^k \mid \min(n, m) \leq k\} \)
(You should describe the PDA’s giving a state transition diagram. You are not required to give the formal description with the table for the transition function.)
[Optional (don’t write anything, just think about this part): what can you say about the language \( L_5 = \{a^n, b^nc^k \mid \max(n, m) \leq k \} \)? Can you give a Push down automaton for it?]

**Problem 4**

Give context free grammars that generates the languages \( L_3, L_4 \) from problem 3.
[Optional (don’t write anything, just think about this part): as in problem 3, do you think the language \( L_5 = \{a^n, b^nc^k \mid \max(n, m) \leq k \} \) can be described by a context free grammar?]