Due: beginning of class on Fri., Feb 25, 2000

**Problem 1**

Let \( L \) be the set of all strings

\[
a^{n_1}b^{n_2}b \ldots a^{n_t}b \in \Sigma^* = \{a, b\}^*
\]

where \( t, n_1, \ldots, n_t \geq 0 \) are positive integers such that \( n_j \neq j \) for some \( j \in \{1, \ldots, t\} \).

(As usual, \( a^{n_i} \) denotes the strings consisting of \( n_i \) copies of the letter “a”.) In this problem you will prove that \( L \) is a context free language. You can break your proof into the following subproblems:

- First prove that the language \( L' = \{c^n d^m e; n + 1 \neq m\} \) over the alphabet \( \{c, d, e\} \) is context free giving a context free grammar that generates \( L' \).

- Then consider \( c, d, e \) as variable symbols and extend the grammar from the first part with rules for \( c, d, e \) in such a way that the new grammar generates the language \( L \).

**Problem 2**

Transform the following PDA into an equivalent grammar.

![Diagram](image)

Figure 1: Push down automaton \( M \)
Problem 3

Let the reverse of a language $L^R$ be as defined in problem 1.24 of the textbook. In this problem you will show that the reverse of a context free language is context free. Answer the following questions:

- Show how to transform any context free grammar $G$ into another grammar $G'$ such that the language generated by $G'$ is the reverse of the language generated by $G$.

- Show how to transform any PDA $M$ into another PDA $M'$ such that the language generated by $M'$ is the reverse of the language generated by $M$.

Problem 4

For each of the following languages says whether the language is context free or not, and prove it.

- $L_1 = \{a^n b^n c^{n+m}|n, m \geq 0\}$
- $L_2 = \{a^n b^{n+1} c^{n+2}|n \geq 0\}$
- $L_3 = \{a^n b^n c^n|n, m \geq 0\}$
- $L_4 = L_1 \cap L_3 = \{a^n b^n c^{2n}|n \geq 0\}$