Answer the following questions:

1. **4 points** Write a deterministic finite automaton to recognize each of the following languages.

   \[ L_1 = \{ w \in \{a, b\}^* | \text{each } a \text{ in } w \text{ is immediately preceded and immediately followed by a } b \} \]

   \[ L_2 = \{ w \in \{a, b\}^* | w \text{ has both } ab \text{ and } ba \text{ as substrings} \} \]
2. **3 points** Show that if \( L \) is a regular language, then so is \( L^R \) where \( L^R = \{ x^R | x \in R \} \). \( x^R \) is the reverse string of the string \( x \). Present a cogent argument outlining the main ideas.

Hint: Make use of nondeterminism.

Since \( L \) is a regular language there is a NFA, say \( A \) accepting it. To obtain \( A_{LR} \) (an automaton accepting \( L^R \)) we proceed as follows:

(a) the states of \( A_{LR} \) are the same as of \( A \)
(b) the alphabet is the same
(c) the transition function is obtained by reversing all arrows in \( A \)
(d) to obtain the initial state of \( A_{LR} \) add a new state and add \( \varepsilon \)-transitions too all states which were final states in \( A \). The newly added state is the initial state of \( A_{LR} \)
(e) there is only one final state, namely the state which was initial state in \( A \)

Why this works:

A word is accepted in \( A_{LR} \) if there is a path from the initial state to the final state. But given the construction this means that in \( A \) there is a path from the initial state to one of the final states. Therefore if \( w \in L(A_{LR}) \) if \( w^R \in L \).

The above argument works the other way around, therefore \( w \in L \) implies \( w^R \in L(A_{LR}) \). This proves that \( L(A_{LR}) = L^R \)

3. **3 points** Design a nondeterministic finite automaton for the following language.
\[ L = \{ x \in \{0,1\}^* | x \text{ has 101 as a substring} \} \]