ALGORITHMS - CSE 101 - SUMMER 1999
Homework 2
Due July 19, 8am in the class. No exception!

Each problem is worth 10 points. We suggest the following steps in writing up your solutions, when they are applicable:

1. Understand the problem. Do not hesitate to contact us if you do not understand what you are supposed to do.
2. Give an informal description (at most 10 lines of English text) of your solution.
3. Write up your design; feel free to use any algorithm presented in the class or in the text book.
4. Analyze the time complexity.
5. Prove formally or informally the correctness of your algorithm.

Problem 1: (k-way Merge Sort)

1. Give an $O(n \log_k k)$ algorithm to merge $k$ sorted lists into one sorted list, where $n$ is the total number of elements in all the input lists. Hint: Use a heap for $k$-way merging. Analyze the time complexity of your algorithm.
2. Analyze a $k$-way merge sort algorithm which first splits the input array in $k$ arrays (instead of 2) of size $n/k$ and then merges them. Is it better than merge sort?

Problem 2: (Blackjack Hand Card Counting)

Give the best algorithm you can for the following problem:

Instance An array $A$ of $n$ positive integers (cards with face values) with values from 1 to $k$, and positive integers $l < n, v < kn$.

Problem Count the number of sets of $l$ array positions (hands of $l$ cards) whose total value is equal to $v$.

Analyze your algorithm in terms of $n, k$ and $l$. Your algorithm should take time polynomial in all 3 parameters.

Problem 3: (3-9, Skiena)

Consider the following data compression technique. We have a table of $m$ text strings, each of length at most $k$. We want to encode a data string $D$ of length $n$ using as few text strings as possible. For example, if our table contains $(a, ba, aab, b)$ and the data string is $baaabaabab$, the best way to encode it is $(b, aab, ba, aab, a)$ - a total of five code words. Give and $O(nmk)$ algorithm to find the length of the best encoding. You may assume that the string has an encoding in terms of the table.