1. (Lecture Question) For all comparison-based sorting algorithms, what is the lower bound complexity in the worst case? (i.e. what is the complexity for the best comparison-based sorting algorithm in the worst case?)

2. In a variant of the game Plinko from the gameshow The Price is Right, the player has one disk which they insert into the top of the board shown below. Each time the disk hits a peg (shown as a black circle), it has a 50% chance of falling to the left, and a 50% chance of falling to the right. Eventually, the disk lands in one of the bins at the bottom of the board. Each bin is marked with a dollar amount, and the player wins the amount of money shown on the bin in which the disk lands.

(a) Suppose that bin 1 is the leftmost bin, and bins are numbered from 1 to 6 reading from left to right. In terms of $i$, what is the probability that the disk falls into bin $i$?

(b) How much money does the player expect to win at this game?

(c) How much money does the player expect to win if you know that the disk falls to the left the first time it hits a peg?

(d) How much money does the player expect to win if you know that the disk falls to the left the first and second time it hits a peg?

3. (a) A teacher randomly calls on students in the class to answer questions. If there are $n$ students in the class, $n$ questions are asked, and each question is asked to one student at random, find the expected number of students that never get called on.
(b) Let $G$ be a simple undirected graph with $n$ vertices and $m$ edges. We partition the vertices of $G$ at random into two sets $L$ and $R$, putting each vertex in $L$ with probability $1/2$, and in $R$ with probability $1/2$, (independently for each vertex). The cut is the set of edges with one endpoint in $L$ and the other in $R$. Find the expected number of edges in the cut.

4. Suppose that $A$ and $B$ are independent events. Prove that $P(A|B) = P(A)$ and explain why this result makes sense.

5. Suppose you have a fair 8-sided die and a fair 15-sided die. How can you use the dice to pick a random integer between 1 and 50 (inclusive) so that each integer is equally likely? Show how you come to your answer.

6. (Lecture Question) What is the minimum number of people who need to be in a room so that the probability that at least two of them have the same birthday is greater than 10%? 30%? 60%?