1. In class we talked about how gates work at the CMOS transistor level. For example, we have analyzed how the transistors composing the two-input NAND gate switch ON/OFF depending on the inputs, and how this determines the value of the output. In this problem you are required to perform a similar analysis for the circuit in the figure below. The circuit is composed by 3 PMOS transistors (P1, P2 and P3) and 3 NMOS transistors (N1, N2 and N3. It has two inputs (a and b) and one output (f). An intermediate signal is also present (c).

![Circuit Diagram]

**a.** In the table below, for each value of the input, write the state (ON/OFF) of each transistor, the value of c and the value of f.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>P1</th>
<th>N1</th>
<th>c</th>
<th>P2</th>
<th>P3</th>
<th>N2</th>
<th>N3</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>on</td>
<td>off</td>
<td>1</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>on</td>
<td>off</td>
<td>1</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>on</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>off</td>
<td>on</td>
<td>0</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>off</td>
<td>on</td>
<td>0</td>
<td>on</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>0</td>
</tr>
</tbody>
</table>

**b.** The circuit is actually the cascade of two gates. Which are they?

Inverter on the left and NOR on the right.

\[ f = a' \text{ Nor } b \]
c. Redraw the circuit using the corresponding gate symbols, and clearly label their input and output.

```
```

2. In class we have seen various theorems that can be used to simplify Boolean functions and reduce them to a more compact form.

Simplify the following Boolean expressions using Boolean algebra. Label each step with the name of the Boolean theorem that you applied.

a. \( x + xx + yz + y'z \)

\[
= x + x + yz + y'z \\
= x + yz + y'z \\
= x + z (y+y') \\
= x + z (1) \\
= x + z
\]

Idempotency
Idempotency
Factoring
Complements
Identity

b. \( (x+y')(xy + xz + yz)(x'y) + xyz + (xz)' \)

\[
= (x+y')(xy + xz + yz)(x+y')' + xyz + (xz)' \\
= 0 + xyz + (xz)' \\
= xyz + x' + z' \\
= yz + xyz + x' + z' + xy \\
= yz + x' + z' + xy \\
= y + yz + x' + z' + xy + y \\
= y + x' + z'
\]

DeMorgan
Complements
DeMorgan
Consensus
Absorption
Consensus
Absorption
3. NAND and NOR gates are universal, which means that you can implement every possible Boolean function with them. Remember that the NOT gate can be implemented using either a NAND or a NOR.

Implement the following functions using only NAND and NOT gates. Do not simplify the functions for this problem.

a. \( (a + b)(c' + d) = (a'b')' (cd')' = (a' \text{ nand } b') (c \text{ nand } d') = (a' \text{ nand } b') \text{ nand } (c \text{ nand } d') \)

\[ a \quad \begin{array}{c}
\longrightarrow \\
\quad b \quad \!
\end{array} \\
\downarrow \\
\quad \begin{array}{c}
\longrightarrow \\
\quad d
\end{array} \\
\downarrow \\
\quad c
\]

b. \( a' + b'c = (a (b'c)')' = a \text{ nand } (b'c)' = a \text{ nand } (b' \text{ nand } c) \)

\[ a \quad \begin{array}{c}
\longrightarrow \\
\quad b
\end{array} \\
\downarrow \\
\quad \begin{array}{c}
\longrightarrow \\
\quad c
\end{array} \\
\downarrow \\
\quad a
\]

Implement the following functions using only NOR and NOT gates.

c. \( (abc)' = a' + b' + c' = ((a' \text{ nor } b') \ ' \text{ nor } c')' \)
d. \(((a + b)'+ c)' + d)' = ((a \text{ nor} b) \text{ nor} c) \text{ nor} d\)

4. Convert the following numbers from decimal to binary
   a. \(83 = (1010011)_b\)
   b. \(25 = (11001)_b\)
   c. \(256 = (100000000)_b\)
   d. \(11 = (1011)_b\)

Convert the following numbers from binary to decimal
   e. \(10010 = (18)_{10}\)
   f. \(1000000001 = (513)_{10}\)
   g. \(111 = (7)_{10}\)
   h. \(1010101 = (85)_{10}\)

Convert the following numbers from binary to hexadecimal
   i. \(1001 = (9)_{16}\)
   j. \(10011 = (13)_{16}\)
   k. \(1111000011110000 = (7878)_{16}\)
   l. \(1111000011110000 = (F0F0)_{16}\)

Convert the following numbers from hexadecimal to decimal
   m. \(ABC = (2748)_{10}\)
5. Design a logical parity checker circuit: The circuit receives 3-bit data input \((a_2, a_1, a_0)\) and produces 1-bit output “\(y\)” which represents the parity of the input data. The output should be equal to zero if the number of ones in the input data is even, and it should be equal to 1 otherwise. For example,

If \((a_2, a_1, a_0) = (1,0,1)\) then \(y=0\)

If \((a_2, a_1, a_0) = (1,0,0)\) then \(y=1\)

Show the truth table, boolean equation for the circuit and draw your circuit.

Don’t simplify the boolean equation. You can use gates with multiple inputs.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

\[ y = a_2'a_1'a_0 + a_2'a_1a_0' + a_2a_1'a_0' + a_2a_1a_0 \]
6. In the following functions, identify and list all the literals, implicants, implicates, minterms and maxterms that appear (note that there may be none).
   a. \( F(a,b,c) = abc + (ab) + a'b + ab'c \)
      Literals: \( a, b, c, a', b' \)
      Implicants: \( abc, ab, a'b, ab'c \)
      Minterms: \( abc, ab'c \)
   b. \( G(a,b,c,d) = (a+b)(a+b+c+d')(a'+b+c+d') \)
      Literals: \( a, b, c, d', a' \)
      Implicants: \( a+b, a+b+c+d', a'+b+c+d' \)
      Maxterms: \( a+b+c+d', a'+b+c+d' \)
7. Given the following truth table, write the corresponding function in terms of SOP and POS for the two outputs X and Y.

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

SOP:

\[
X = a'bc + ab'c' \\
Y = a'b'c' + a'bc' + ab'c' + abc' + abc
\]

POS:

\[
X = (a+b+c)(a+b+c')(a+b'+c)(a'+b+c)(a'+b'+c') \\
Y = (a+b+c')(a+b'+c')(a'+b+c')
\]