## Homework 5

### 5.1 [10 points]
Derive closed-form expressions for the following recursive formulas:

(a) \( a_n = 2a_{n-1} + 4, a_0 = 2 \)

(b) \( a_n = -3a_{n-1} + 3, a_0 = 5 \)

### 5.2 [15 points]
Derive closed-form expressions for each of the following recursive formulas:

(a) \( a_n = 10a_{n-1} - 25a_{n-2}, a_0 = 1, a_1 = 2 \)

(b) \( a_n = 5a_{n-1} - 6a_{n-2}, a_0 = 2, a_1 = 3 \)

### 5.3 [20 points]
In the dice-game of craps a bet on the passline is resolved in the following way. The game consists of repeatedly rolling two 6-sided dice a number of times. The first roll is called the come out roll. If the number comes up 2, 3 or 12 the player immediately loses the passline bet. If the number comes up 7 or 11 the player immediately wins the bet. If the roll is any other number that number is called the point (e.g. the point may be 5). The dice are then repeatedly rolled until either a 7 comes up (in which case the player loses) or the point comes up again (in which case the player wins).

(a) Compute the probability that given a point is established (i.e. the game doesn’t end on the come out roll) that the player will win

(b) Compute the overall probability that the player will win the pass line bet.
5.4 [15 points]
How many sequences of binary digits of length \( n \) can be made such that the sequence ‘10’ never occurs.

5.5 [15 points]
A casino game allows the player to win his or her original bet with probability .49 and lose his or her bet with .51 probability. Suppose a player implements the following betting strategy. Start by betting $1. If win, then quit. If lose and total number of bets so far is less than \( n \), then double the bet, otherwise quit.

(a) Show that the probability of the player winning $1 using this strategy approaches 1 as the limit on the number of bets gets higher and higher.

(b) Provide a formula for the expected value of this betting strategy as a function of the bet limit \( n \). Do you think this is a good betting strategy?

5.6 [10 points]
How many unique ways can a strip of squares of size \( n \times 1 \) be tiled using the following types of tiles:

1. Red 1 square by 1 square tiles
2. Blue 1 square by 1 square tiles
3. Green 1 square by 1 square tiles
4. Black 1 square by 2 square tiles

Assume there are infinite quantities of each type of tile available to you.