### Homework 4

#### 4.1 [15 points]
Team A and Team B are competing to have the best record in their conference. Team A’s record is 20-10 while Team B’s is 18-12. Each team has 4 games left in its season. Assuming that team A and B are not playing each other and that the probability of each team winning each of its last four games is $\frac{3}{5}$ (with the outcome of all games independent), what is the probability that Team B will finish with a better record than team A?

#### 4.2 [10 points]
Let $X$ be a random variable representing the number of Clubs in a 10 card poker hand and $Y$ be a random variable representing the number of Diamonds. What is $E[X + Y]$?

#### 4.3 [10 points]
4 boys each select independently and uniformly at random one of 3 girls to throw a frisbee to. Let $X$ represent the number of frisbees that the first girl gets. What is $E[X]$? What is $\text{Var}[X]$?

#### 4.4 [15 points]
Given an image of a face, you observe that you can see the teeth of the person in the image ($TS = \text{teeth showing}$) and that their brows are furrowed ($BF = \text{brows furrowed}$). Assuming that there are only two types of facial expressions joyful and angry and that: $P(\text{joyful}) = .6$, $P(\text{angry}) = .4$, $p(TS|\text{angry}) = .5$, $P(BF|\text{angry}) = .6$, $P(TS|\text{joyful}) = .8$, and $P(BF|\text{joyful}) = .05$.

Additionally assume that $TS$ and $BF$ are conditionally independent given the expression (angry or joyful). What is the probability that the image of the face in the picture you see is of someone who is angry? In other words, what is $P(\text{angry}|BF \cap TS)$?

Hint: by Bayes’ rule: $P(\text{angry}|BF \cap TS) = \dfrac{P(BF\cap TS|\text{angry})P(\text{angry})}{P(BF\cap TS)}$. 

4.5  [15 points]
10 people are each asked to randomly select an integer between 1 and 10 inclusive and write it on a piece of paper. Assume that each person selects their number independently and uniformly at random. Let $X$ be the number of numbers 1 through 10 that are not selected by any of the 10 people. What is $E[X]$?

4.6  [15 points]
A fair coin is flipped 3 times. We define the following random variables:

1. $X_1$ is 1 if the first coin came up heads, 0 otherwise
2. $X_2$ is 1 if the second coin came up heads, 0 otherwise
3. $X_3$ is 1 if the third coin came up heads, 0 otherwise
4. $Y_1$ is 1 if the first two coins came up heads, 0 otherwise
5. $Y_2$ is 1 if the last two coins came up heads, 0 otherwise

(a) Show that $X_1$ and $X_2$ are independent random variables
(b) Show that $Y_1$ and $Y_2$ are not independent random variables