Homework 3

3.1 [10 points]
How many permutations on the set $S = \{1, 2, 3, 4, 5, 6\}$ have order 6?
(Hint: remember that the order of a permutation is the least common multiple of its cycle lengths)

3.2 [15 points]
20 identical iPhones and 10 identical Android phones are being distributed to 10 computer science and 2 math majors. How many ways can the phones be allocated if:

(a) There are no restrictions on the allocation
(b) Each computer science major gets exactly 1 iPhone
(c) Each math major gets at least 2 Android phones
(d) Each math major gets no more than 2 iPhones

3.3 [15 points]
8 boys each select independently and uniformly at random one of 5 girls to throw a frisbee to (the frisbees are not considered distinct, while the boys and girls of course are):

(a) How many allocations of the 8 frisbees to the 5 girls are possible such that every girl gets at least one frisbee (remember frisbees are not considered distinct)? By allocation we just mean how many frisbees each girl receives, not which boy threw it to them.

(b) What is the probability that each of the girls receives at least one frisbee? (It may be helpful to explicitly specify your sample space and probability function)
3.4 [10 points]
Given a 52 card deck, 40 cards are selected without replacement. Show that the probability of selecting the Ace of Clubs on the first draw is equal to the probability of selecting it on the 40th draw.

3.5 [15 points]
An urn is initially filled with 3 green marbles, 2 red marbles, and 4 blue marbles. Three marbles are selected without replacement. What is the probability that:
(a) All marbles have different colors
(b) All marbles are the same color
(c) The third marble selected is red

3.6 [15 points]
An urn is initially filled with 3 green marbles, 2 red marbles, and 4 blue marbles. First, one marble is selected. If it is red it is put back in the urn along with 4 additional red marbles. If it is blue, then the blue marble is put back in the urn along with 3 additional green marbles. If it is green then the marble is removed from the urn and no additional marbles are placed in the urn. Next, a second marble is selected. What is the probability that both marbles are the same color?