Homework 2

2.1  [15 points]
Count the number of each of the following 6-card hands that can be made from the standard 52 card deck:

(a) A hand consisting of 3 pairs (the ranks of the three pairs are not necessarily distinct. So a 4 of a kind and a pair would count)

(b) A hand consisting of a four of a kind and a pair

(c) A hand consisting of a three pairs each with a distinct rank

(d) A hand consisting of at least 5 cards of the same suit

2.2  [15 points]
Let \( S \) be the cartesian product \( \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\}^{10} \). Recall that the elements of \( S \) will be lists of length 10 composed of the elements in the set \( \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\} \). How many elements of \( x \in S \) have at least one element from the set \( \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F\} \) that appears at least 3 times?

2.3  [10 points]
How many ways can 20 red hats and 30 blue hats be distributed to 100 people such that each person gets at most one hat? Hats of the same color are not considered distinct.
2.4 Consider functions from the set \( S = \{0, \ldots, 9\} \) to itself. Count the number of functions that have the following properties:

(a) No restrictions
(b) Surjective
(c) Injective
(d) Bijective

2.5 Calculate (as a function of \( n \)) the proportion of functions from the set \( S_n = \{1 \ldots n\} \) to itself that are permutations.

2.6 A dice is rolled 100 times and the results are recorded as a sequence of elements from the set \{1 \ldots 6\}. How many sequences are possible that include at least 1 five?

2.7 [10 points]
Prove using the technique of double counting that for \( n, m, r \) all in the set of natural numbers that \( \binom{n+m}{r} = \sum_{k=0}^{r} \binom{m}{k} \binom{n}{r-k} \)