CSE 21: Midterm 2
May 25, 2006

No books, no calculators. One 8.5x11 page of handwritten notes.

Name: ____________________________

Student ID:_______________________

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1. 10 pts. Suppose that a bank pays 6% interest on money during the first year it is in the account and 8% on money that is in the account for more than one year. Interest is paid at the end of each year (compounded annually). Assume $1000 are deposited in such an account.

At the end of the first year, the account will be worth $1060, since it will have earned $60 on the initial $1000. After two years, it will be worth $1143.60: 8% was earned on the initial $1000 that’s been in the account more than one year, and then 6% was earned on the $60 that’s been in the account for only one year.

Give (but do not solve) a recurrence relation for the amount of money in the account after \( n \) years.
2. 20 pts. Solve the recurrence relation

\[ a_n = 2a_{n-1} + 15a_{n-2} \]

\[ a_0 = 1 \]

\[ a_1 = 13 \]
3. 15 pts. An urn contains 2 red marbles and 3 green marbles. A marble is chosen randomly from the urn.

- If the marble is red, a fair coin is tossed: if it comes up heads, 1 red marble is added to the urn; if it comes up tails, 3 green marbles are added to the urn.
- If the marble is green, two green marbles are added to the urn.

Finally, a second marble is chosen from the urn.

(a) What is the probability that the second marble is red?

(b) If the second marble is red, what is the probability that the first marble was red?

(c) If the first marble was red, what is the probability that the coin toss came up heads.
4. 15 pts. Two fair dice are thrown: one green, and one red.

(a) What is the probability space?

(b) What is the probability that the green die shows a six or the total on both dice is greater than 10?
5. 20 pts. Solve the following problem using generating functions. How many ways are there to choose 10 DVDs from six different *Lost* DVDs and 8 identical copies of *American Idol*?
6. 20 pts. Give a solution to the following recurrence relation (to within a constant bound): (Hint: use a change of variable: $m = \lg n$.)

\[
\begin{align*}
a(n) &= 3a(\sqrt[3]{n}) + \lg n \\
a(1) &= 1
\end{align*}
\]
7. **15 pts. Extra Credit.** Someone offers to play a game with you. You flip a coin until a tail comes up. The total number of flips, $n$, determines the prize, $2^n$.

For example, if you flip a tail right off, you win $2$. If you flip a head, then a tail, you win $4$. Two heads and then a tail will win you $8$.

What is the expected number of dollars you win by playing this game?