CSE21 (summer II 2006)

Midterm 2

Write down your problem solving process and how you get to the solution. No points are given without explanation.

Name:

ID #:

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1. Four-pole Hanoi Tower: Suppose that the Tower of Hanoi problem has four poles in a row instead of three. Rings can be transferred one by one from one pole to any other pole, but at no time may a larger ring be placed on top of a smaller ring. Let $s(n)$ be the minimum $\#$ of moves needed to transfer the entire tower of $n$ rings from the left-most pole to the right-most pole (20 points)
   a) Find $s(1)$, $s(2)$ and $s(3)$

b) Explain how to do $s(4)$

c) Find the recurrence representation of $s(n)$

d) Find $s(6)$
2. Counting strings (20 points)
   a) Make a list of all bit strings of lengths one, two, three and four that do not contain the bit pattern 000
   
   b) For each non-negative integer n, let \( d(n) \) = the \# of bit strings of length n that do not contain the bit pattern 000, find \( d(0) \) to \( d(4) \)
   
   c) Find a recurrence relation for \( d(n) \)
   
   d) What is \( d(6) \)?
3. Suppose for each step you take when climbing a staircase, you can move up either one or two stairs. To climb the entire staircase you can take one stair at a time or two at a time or any combination of them. Let $S(n)$ be the number of different ways to climb a staircase with $n$ stairs: (15 points)
   a) Find $S(1)$, $S(2)$ and $S(3)$

   b) Find a recurrence relation for $S(n)$

   c) What is $S(6)$?
4. Solve recursion by iteration (15 points)
   a) \( h(k) = 2^k + h(k-1); h(0) = 1 \) (hint: by iteration)

   b) \( b(k) = 7b(k-1) - 10b(k-2); b(0) = 2, b(1) = 2 \) (hint: solving linear recursion)

   c) \( T(n) = 2T(n/2) + 5 \) (hint: by master method)
5. Show that for any real number $x > 1$, (10 points)
   a) $|x^2| \leq |2x^2 + 15x + 4|$

b) Use the $\Omega$ or $O$ notation to express the results of (a)

6. Use the definition of $\Theta$-notation to show that $5x^3 + 72x^2 + 30$ is $\Theta(x^3)$ (10 points)
7. Determine the following f(n) is Θ(?) (15 points)
   a) \( f(n) = \frac{n(n + 2)(3n + 4)}{6n - 2} \)
   
   b) \( f(n) = 5n^3 + \frac{1}{n^4 + 1} + \log n \)
   
   c) \( f(n) = \frac{\sqrt{n(n - 4)}}{n^5 - 2} \)

8. For the following algorithm segment with input size n: (15 points)
   for i:=1 to 2n
     for j:=1 to n
       for k:=1 to j
         a:= i*j+k
       next k
     next j
   next i
   a) Compute the actual number of operations (operation within the innermost loop) that must be performed when the algorithm is executed.
   
   b) Find an order for the algorithm segment.