A class has 10 men and 20 women of which half the men and 1/4 of the women are over 25 years old. What is the probability \( p \) that a person chosen at random is a man or is over 25?

Let

A be an event that a man is chosen, and
B be an event that a person who is over 25 years is chosen.

Then,

\[
P(A) = \frac{10}{30}
\]

\[
P(B) = \frac{20}{30} \cdot \frac{1}{4} + \frac{10}{30} \cdot \frac{1}{2} = \frac{10}{30}
\]

(For \( P(B) \), \( \frac{20}{30} \cdot \frac{1}{4} \) is the number of men who are over 25, and \( \frac{10}{30} \cdot \frac{1}{2} \) is the number of women who are over 25.)

Therefore,

\[
P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{10}{30} + \frac{10}{30} - \left( \frac{10}{30} \cdot \frac{1}{2} \right) = \frac{15}{30} = \frac{1}{2}
\]

In an exam there’re 10 multiple-choice problems, each problem with choices A - E. A clueless student decides to throw random answers

a. Suppose for each problem, if answered correctly you get 10 points, otherwise 0 point. What is the expected score of the clueless student?

For each problem, the expected value of a score is

\[
\frac{1}{5} \cdot 10 + \frac{4}{5} \cdot 0 = 2
\]

Hence, for 10 problems,

\[
2 \cdot 10 = 20
\]

b. If for each problem, answered correctly you get 10 points, otherwise -2 points. What is the expected score of the clueless student then?
For each problem, the expected value of a score is

\[
\frac{1}{5} \times 10 + \frac{4}{5} \times (-2) = \frac{2}{5}
\]

Hence, for 10 problems,

\[
\frac{2}{5} \times 10 = 4
\]

A fair coin is tossed until either counted 2 heads or 3 tails. What is the expected number of tosses?

By drawing a tree diagram, we find there are 6 cases for 4 tosses with each probability \(\frac{1}{16}\), 3 cases for 3 tosses with each probability \(\frac{1}{8}\), and 1 case for 2 tosses with a probability \(\frac{1}{4}\). Based on them, the expected number of tosses is,

\[
4 \times 6 \times \frac{1}{16} + 3 \times 3 \times \frac{1}{8} + 2 \times 1 \times \frac{1}{4} = \frac{25}{8}
\]