Answer key for Quiz 3

Binomial problem: when the expression \((a + b)^4\) is multiplied out, terms of the form \(aaaa, abaa, baba, bbba\), and so on are obtained. Consider the set \(S\) of all strings of length 4 over \(\{a, b\}\).

a. What is \(|S|\)? In other words, how many strings of length 4 can be constructed using a’s and b’s?

There are 4 places in a string. For each place, we have 2 choice, \(a\) or \(b\). Therefore,

\[
2 \times 2 \times 2 \times 2 = 2^4.
\]

b. How many strings of length 4 over \(\{a, b\}\) have three a’s and one b?

Since we need to choose 1 place for B from 4 places,

\[
\binom{4}{1} = 4
\]

c. How many strings of length 4 over \(\{a, b\}\) have two a’s and two b’s?

Since we need to choose 2 place for B from 4 places,

\[
\binom{4}{2} = \frac{4 \times 3}{2 \times 1} = 6
\]

Bars & stars problem: how many solutions there are to the equation:

a. \(x_1 + x_2 + x_3 = 20\), each \(x_i\) is a nonnegative integer

We can think of this problem as separating 20 objects into 3 types. And, the number of 'stars' is 20 and the number of 'bars' is 3-1=2.

\[
\binom{20 + 3 - 1}{20} = \binom{22}{20} = \binom{22}{2} = \frac{22 \times 21}{2 \times 1}
\]
b. \( x_1 + x_2 + x_3 = 20 \), each \( x_i \) is a positive integer.

Since \( x_i \) is positive, each of \( x_1 \), \( x_2 \), and \( x_3 \) is at least 1. So, at first, we remove 3 from 20 for that. Then, the number of 'stars' is 17 and the number of 'bars' is 3-1=2.

\[
\binom{17 + 3 - 1}{17} = \binom{19}{17} = \binom{19}{2} = \frac{19 \times 18}{2 \times 1}
\]