Answer key for Quiz 1 (individual quiz)

* I use ‘*’ for representing multiplication, ‘u’ for a union as in (|A₁| u |A₂|), and ‘n’ for an intersection as in (|A₁| n |A₂|).

Write down your problem solving process and how you get to the solution. No points is given without explanation.
Name:
ID #:

License plate problem: the license plate in California has one digit followed by three letters then by three digits.

a. How many different license plates are possible? (no need to calculate out the final number)

Let the first step be to choose a digit to put in position 1, let each steps 2-4 be to choose a letter to put in positions 2-4, and let each steps 5-7 be to choose a digit to put in positions 5-7. Since there are 10 digits (0-9) and 26 letters, the number of license plates is

\[10*26*26*26*10*10*10 \text{ (or } 10^4 * 26^3)\]

b. How many license plates could contain CAR as the three letters?

In this case, there only one way to perform step 2-4 (because the three letters must be CAR in this order), the number of license plates is

\[10*10*10*10 \text{ (or } 10^4)\]

c. How many license plates are possible in which all the digits are distinct?

In this case, there are 10 ways to perform step 1, 9 ways to perform step 5, 8 ways to perform step 6, and 7 ways to perform step 7. Therefore, the number of license plates is

\[10*26*26*26*9*8*7\]

Inclusion / Exclusion problem: How many integers from 1 through 100000 contain each of the digits 1, 2, and 3 at least once? (hint: for each \(i=1, 2, \text{ and } 3\), let \(A_i\) be the set of all integers from 1 through 100000 that do not contain the digit \(i\))

For each \(i=1, 2, \text{ and } 3\), let \(A_i\) be the set of all integers from 1 through 100000 that do not contain the digit \(i\). Then, we want to compute \(|U| - (|A_1| u |A_2| u |A_3|)\), which is a dark grey (background) part in the diagram.
By the inclusion/exclusion rule,

\[ |A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - (|A_1| \cap |A_2|) - (|A_1| \cap |A_3|) - (|A_2| \cap |A_3|) + (|A_1| \cap |A_2| \cap |A_3|). \]

Now, put the number 100000 aside for a while, imagine each integer 1 through 99999 as a string of five digits with leading 0’s allowed. Then,

\[ |A_1| = 9^5 - 1 \]

(9 digits except 1 can be used for each of five positions. 00000 should be removed. 100000 should not be included since it has 1.)

\[ |A_2| = 9^5 \]

(in a similar manner, but the case of 10000 is added.)

\[ |A_3| = 9^5 \]

(same as above)

\[ (|A_1| \cap |A_2|) = 8^5 - 1 \]

(8 digits can be used for each of five positions. 00000 should be removed. 100000 should not be included.)

\[ (|A_1| \cap |A_3|) = 8^5 - 1 \]

(same as above)

\[ (|A_2| \cap |A_3|) = 8^5 \]

(in a similar manner, but the case of 10000 is added.)

\[ (|A_1| \cap |A_2| \cap |A_3|) = 7^5 - 1 \]

(7 digits can be used for each of five positions. 00000 should be removed. 100000 should not be included.)

Thus,

\[ |A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - (|A_1| \cap |A_2|) - (|A_1| \cap |A_3|) - (|A_2| \cap |A_3|) + (|A_1| \cap |A_2| \cap |A_3|) = (9^5 - 1) + 9^5 + 9^5 - (8^5 - 1) - (8^5 - 1) - 8^5 + (7^5 - 1). \]

and so

\[ |U| - (|A_1 \cup A_2 \cup A_3|) = 100000 - \{9^5 - 1 + 9^5 + 9^5 - (8^5 - 1) - (8^5 - 1) - 8^5 + (7^5 - 1)\}. \]