

Quine-McCluskey Tableaux Reduction Rule

Definition 1

Two rows a and b of a reduced prime table, which cover the same minterms are said to be **interchangeable**.

Definition 2

Given two rows a and b in a reduced prime implicant table, row a is said to **dominate** row b if row a has checks in all the columns in which row b has checks and rows a and b are not interchangeable.

Definition 3

Two columns c and d of a reduced prime table, which are covered by the same prime implicants are said to be **interchangeable**.

Definition 4

Given two columns c and d in a reduced prime implicant table, column c is said to **dominate** column d if column c has checks in all the rows in which column d has checks and column c and d are not interchangeable.

Theorem 1

Let a and b be rows of a reduced prime implicant table. If a dominates b or a and b are interchangeable, there exists a minimal sum of products that does not include b .

Theorem 2

Let c and d be columns of a reduced prime implicant table. If c is dominated by d or c and d are interchangeable, there exists a minimal sum of products that does not include d .

Example

Determine the minimal sum-of-products form for

$$F(A,B,C,D,E) = \Sigma(1,2,3,5,9,10,11,18,19,20,21,23,25,26,27)$$

Step 1:

Using **tabulation method**, generating all the **prime implicants** and construct a prime implicant table, as shown in Table 1.

Name	Expression	1	2	3	5	9	10	11	18	19	20	21	23	25	26	27
P ₁	C'D		X	X			X	X	X	X					X	X
P ₂	BC'E					X		X						X		X
P ₃	A'C'D	X		X		X		X								
P ₄	A'B'D'E	X			X											
P ₅	B'CD'E				X							X				
P ₆	AB'CD'										X	X				
P ₇	AB'DE									X			X			
P ₈	AB'CE											X	X			

Table 1

Step 2:

Based on the information in Table 1, select all the **essential prime implicants**. As shown in Table 2, the EPIs (marked by a preceding *) are

C'D (P₁), BC'E (P₂), AB'CD' (P₆)

Name	Expression	1	2	3	5	9	10	11	18	19	20	21	23	25	26	27
*P ₁	C'D		X	X			X	X	X	X					X	X
*P ₂	BC'E					X		X						X		X
P ₃	A'C'D	X		X		X		X								
P ₄	A'B'D'E	X			X											
P ₅	B'CD'E				X							X				
*P ₆	AB'CD'										X	X				
P ₇	AB'DE									X			X			
P ₈	AB'CE											X	X			

Table 2

Step 3:

Reduce the prime implicant table by crossing out the minterms already covered by the implicants selected (applying **Theorem 2**). As shown in Table 3, besides m₂, m₁₀, m₁₈, m₂₅ and m₂₆, the minterm m₃, m₉, m₁₁, m₁₉, m₂₁, and m₂₇ are also be covered by the P₁, P₂ or P₃. Therefore they are crossed out.

Name	Expression	1	2	3	5	9	10	11	18	19	20	21	23	25	26	27
*P ₁	C'D		X	X			X	X	X	X					X	X
*P ₂	BC'E					X		X						X		X
P ₃	A'C'D	X		X		X		X								
P ₄	A'B'D'E	X			X											
P ₅	B'CD'E				X							X				
*P ₆	AB'CD'										X	X				
P ₇	AB'DE									X			X			
P ₈	AB'CE											X	X			

Table 3

Step 4:

In the reduced prime table, P_4 dominates P_3 and P_5 , P_7 and P_8 are interchangeable. Therefore, there is a minimal cover which does not include P_3 , P_5 and P_8 , as in Table 4 (applying **Theorem 1**).

Name	Expression	1	2	3	5	9	10	11	18	19	20	21	23	25	26	27
* P_1	$C'D$		X	X			X	X	X	X					X	X
* P_2	$BC'E$					X		X						X		X
P_3	$A'C'D$	X		X		X		X								
P_4	$A'B'D'E$	X			X											
P_5	$B'CD'E$				X							X				
* P_6	$AB'CD'$										X	X				
P_7	$AB'DE$									X			X			
P_8	$AB'CE$											X	X			

Table 4

Finally the minimal cover is:

$$F = C'D + BC'E + A'B'D'E + AB'DE$$

or $F = C'D + BC'E + A'B'D'E + AB'CE$