Problem Set 3

Problems 1-3 pertain to the material covered during the fourth week of class (July 19 - July 23). The remaining problems pertain to the material covered during the fifth week (July 26 - July 30).

1. Prove that the following language is co-recognizable:
   \[ A = \{ \langle G_1, G_2 \rangle \mid G_1, G_2 \text{ are CFGs and } L(G_1) = \overline{L(G_2)} \} \]

2. Prove that the following language is co-recognizable:
   \[ B = \{ \langle G, G_1, G_2 \rangle \mid G, G_1, G_2 \text{ are CFGs and } L(G) = L(G_1) \cap L(G_2) \} \]

3. Let \( L \) be a recognizable language that is not decidable. Prove that for any TM \( M \) that recognizes \( L \) there are infinitely many strings on which \( M \) does not halt.

4. Prove that the following language is undecidable:
   \[ A = \{ \langle G_1, G_2 \rangle \mid G_1, G_2 \text{ are CFGs and } L(G_1) = \overline{L(G_2)} \} \]

5. Prove that the following language is undecidable:
   \[ B = \{ \langle G, G_1, G_2 \rangle \mid G, G_1, G_2 \text{ are CFGs and } L(G) = L(G_1) \cap L(G_2) \} \]

6. Prove that the following language is undecidable:
   \[ C = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs and } M_1(\varepsilon) \text{ halts and } M_2(\varepsilon) \text{ does not halt} \} \]

7. Prove that the following language is undecidable:
   \[ D = \{ \langle M \rangle \mid M \text{ is a TM that accepts at most one string which ends in a } 0 \} \]

8. Is the following language decidable or undecidable? Justify your answer (i.e., if \( E \) is decidable then construct a decider for it; otherwise, prove that it is undecidable).
   \[ E = \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs and } L(M_1) \subseteq L(M_2) \} \]