Administrative Issues

- Today is the last day to request a regrade!
- Deadline to submit TRUE/FALSE and/or multiple choice questions for the final exam is W, July 28.
- Review for Test 2 on F, July 23, 11:00 AM – 12:20 PM in HSS 1330

Lecture 15 Overview

- TRUE/FALSE questions about decidability and recognizability
- Decidable Problems
TRUE/FALSE Questions

1) If there is a TM that does not halt when given input a string in a language \( L \) then \( L \) is not decidable.

FALSE.

Let \( L = \Sigma^* \). Then \( L \) is decidable (a TM that always accepts decides it), but the following TM does not halt when given input a string in \( L \):

\[ M = \text{On input a string } w \]

\[ \text{While } (0 = 0) \text{ write } 0 \text{ and move } \text{R} \]

TRUE/FALSE Questions

2) If there is a TM that does not halt when given input a string in a language \( L \) then \( L \) is not recognizable.

FALSE.

The same example used for question 1) justifies this.
TRUE/FALSE Questions

3) If $L$ is regular then $L$ is decidable.

TRUE.

Let $L$ be a regular language. Then there is a DFA $D$ that recognizes $L$. The following TM decides $L$:

$M = \text{On input a string } w$

  Simulate $D$ on input $w$
  If the simulation ends in an accept state, then
    \textit{accept}
  else \textit{reject} EndIf

TRUE/FALSE Questions

4) If $L$ is a CFL then $L$ is decidable.

TRUE.

Let $L$ be a CFL. Then there is a PDA $P$ that recognizes $L$. \textbf{The following TM decides $L$}: NO!

$M = \text{On input a string } w$

  Simulate $P$ on input $w$
  If the simulation ends in an accept state, then
    \textit{accept}
  else \textit{reject} EndIf
TRUE/FALSE Questions

Let \( L \) be a CFL. Then there is a PDA \( P \) that recognizes \( L \). The following TM decides \( L \): NO!

\[ M = \text{On input a string } w \]
\[ \quad \text{Simulate } P \text{ on input } w \]
\[ \quad \text{If the simulation ends in an accept state, then} \]
\[ \quad \quad \text{accept} \]
\[ \quad \text{else reject} \]

EndIf

Some branches of \( P \)'s computation might not halt. For example, some branch may push symbols onto the stack forever. The TM \( M \) would not be a decider.

TRUE/FALSE Questions

4) If \( L \) is a CFL then \( L \) is decidable.

The statement is TRUE, but we have to use a different argument to justify it.
TRUE/FALSE Questions

5) If $L$ is finite then $L$ is decidable.

TRUE.

Let $L = \{w_1, \ldots, w_n\}$. The following TM decides $L$:

$M =$ On input a string $w$
If $w = w_1$ or $\cdots$ or $w = w_n$ then accept
else reject EndIf

Alternative justification:
If $L$ is finite then $L$ is regular. By 3), $L$ is decidable.

TRUE/FALSE Questions

6) If $L$ is infinite then $L$ is undecidable.

FALSE

Let $L = \Sigma^*$. Then $L$ is infinite and it is decidable.
TRUE/FALSE Questions

7) If there is a TM that halts when given input a particular string in a language $L$ then $L$ is decidable.

FALSE

Let $L$ be an undecidable language (we know such languages exist). The following TM halts when given input a particular string in $L$:

$M = \text{On input a string } w$

accept

TRUE/FALSE Questions

8) If there is a TM that halts when given input any string in a language $L$ then $L$ is decidable.

FALSE

The same example used for question 7) justifies this.
TRUE/FALSE Questions

9) If there is a TM that rejects some string in a language $L$ then $L$ is not recognizable.

FALSE

Let $L = \Sigma^*$. Then $L$ is recognizable, but the following TM rejects a string in $L$.

$M = \text{On input a string } w$

reject

TRUE/FALSE Questions

10) If $L$ is recognizable then $\overline{L}$ is recognizable.

FALSE

There are languages $L$ such that $L$ is recognizable but $\overline{L}$ is not recognizable. We will discuss examples of such languages next week.
TRUE/FALSE Questions

11) If $L$ is recognizable and $\overline{L}$ is recognizable then $L$ is decidable.

TRUE

Let $L$ be a recognizable language such that $\overline{L}$ is recognizable. Then there are TMs $M$ and $\overline{M}$ that recognize $L$ and $\overline{L}$, respectively. The following TM decides $L$:

$D = \text{On input a string } w$
Run $M(w)$ and $\overline{M}(w)$ in parallel
If $M(w)$ accepts then $accept$ EndIf
If $\overline{M}(w)$ accepts then $reject$ EndIf
TRUE/FALSE Questions

11) A NTM is a decider if and only if all branches of computation halt on all inputs.

TRUE

By definition.

What is wrong with the statement?

TM $M$ rejects language $L$.

It is meaningless. We have not defined (nor will we) what it means for a language to be rejected by a TM (or a FA or a PDA). We have only defined what it means for a STRING to be rejected by a TM.