1. In a certain town, 25% of the citizens drive red cars, 45% of the citizens have received a speeding ticket, and 20% have both received a speeding ticket and drive red cars. A citizen is selected at random.

(a) If she drives a red car, what is the probability that she has received a speeding ticket?

\[ P(S \mid R) = \frac{P(S \cap R)}{P(R)} = \frac{0.20}{0.45} = \frac{4}{9} = 80\% \]

(b) What is the probability that she drives a red car if she did not receive a speeding ticket?

\[ P(R \mid S^c) = \frac{P(R \cap S^c)}{P(S^c)} = \frac{P(R) - P(R \cap S)}{P(S^c)} = \frac{0.45 - 0.20}{1 - 0.45} = \frac{0.25}{0.55} = \frac{25}{55} = 0.45 = 45\% \]

2. There are two urns with the following initial distribution:

- Urn A contains 7 red marbles and 3 white marbles
- Urn B contains 2 red marbles and 4 white marbles

First, one marble is drawn from A and placed into B.
Second, another marble is drawn from A and placed into B.
Third, one marble is drawn from B.

(a) What is the probability that all three marbles drawn were the same color?

Let the notation A(7:2) indicate that there are 7 reds and 2 whites in urn A.

Case 1: Three Reds

Initially A(7:3) and B(2:4). Red is chosen from A with P(R) = \( \frac{7}{10} \) and placed into B
Now A(6:3) and B(3:4). Red is chosen from A with P(R) = \( \frac{6}{10} \) and placed into B
Now A(5:3) and B(4:4). Red is chosen from B with P(R) = \( \frac{4}{8} \)

Case 2: Three Whites

Initially A(7:3) and B(2:4). White is chosen from A with P(R) = \( \frac{3}{10} \) and placed into B
Now A(7:2) and B(2:5). White is chosen from A with P(R) = \( \frac{2}{10} \) and placed into B
Now A(7:1) and B(2:6). White is chosen from B with P(R) = \( \frac{4}{8} \)

Answer: \( P(\text{RRR}) + P(\text{WWW}) = \left( \frac{7}{10} \right) \left( \frac{3}{4} \right) \left( \frac{2}{6} \right) + \left( \frac{3}{10} \right) \left( \frac{1}{3} \right) \left( \frac{1}{6} \right) = \left( \frac{21}{60} \right) + \left( \frac{3}{60} \right) = \left( \frac{24}{60} \right) = \frac{24}{50} = 28\% \)

(b) What is the probability of drawing at most two reds if the first marble drawn is red?

Answer: \[ \frac{P(\text{RWR}) + P(\text{RWW}) + P(\text{RWW})}{P(R ? ?)} = \left[ \left( \frac{7}{10} \right) \left( \frac{3}{8} \right) \left( \frac{3}{5} \right) + \left( \frac{7}{10} \right) \left( \frac{3}{8} \right) \left( \frac{2}{5} \right) + \left( \frac{7}{10} \right) \left( \frac{2}{8} \right) \left( \frac{3}{5} \right) \right] \div \left( \frac{7}{10} \right) \]

\[ = \left( \frac{3}{8} \right) \left( \frac{3}{5} \right) + \left( \frac{3}{8} \right) \left( \frac{2}{5} \right) + \left( \frac{2}{8} \right) \left( \frac{3}{5} \right) = \left( \frac{9}{25} \right) + \left( \frac{6}{25} \right) + \left( \frac{6}{25} \right) = \frac{21}{25} = 84\% \]

\[ \frac{P(\text{RWR}) + P(\text{RWW}) + P(\text{RWW})}{P(R ? ?)} = \left[ \left( \frac{7}{10} \right) \left( \frac{3}{8} \right) \left( \frac{3}{5} \right) + \left( \frac{7}{10} \right) \left( \frac{3}{8} \right) \left( \frac{2}{5} \right) + \left( \frac{7}{10} \right) \left( \frac{2}{8} \right) \left( \frac{3}{5} \right) \right] \div \left( \frac{7}{10} \right) \]

\[ = \left( \frac{3}{8} \right) \left( \frac{3}{5} \right) + \left( \frac{3}{8} \right) \left( \frac{2}{5} \right) + \left( \frac{2}{8} \right) \left( \frac{3}{5} \right) = \left( \frac{9}{25} \right) + \left( \frac{6}{25} \right) + \left( \frac{6}{25} \right) = \frac{21}{25} = 84\% \]
3. Consider a 5-day weather pattern where it is equally likely to be rainy as it is to be sunny
Let A = event that three or more days are rainy.
Let B = event that the first two days are sunny.
Let C = event that all days have the same weather.

(a) Find the probability of each event.

\[ P(A) = \frac{P(3R) + P(4R) + P(5R)}{\text{All Possible Weather}} = \frac{{3 \choose 3} + {3 \choose 4} + {3 \choose 5}}{\frac{1}{32}} = \frac{10 + 5 + 1}{32} = \frac{1}{2} \]

\[ P(B) = P(RR? ??) = \left( \frac{1}{2} \right)^2 \frac{1}{2} = \frac{1}{4} \]

\[ P(C) = P(5R) + P(5S) = \left( \frac{1}{32} \right) + \left( \frac{1}{32} \right) = \frac{1}{16} \]

(b) For each pair of events, decide whether they are independent. Defend your answer.

NO: \[ P(A \cap B) \neq P(A) \cdot P(B) \]
\[ A \cap B = \{ SSRRR \} \implies P(A \cap B) = \frac{1}{32} \]
\[ P(A) \cdot P(B) = \left( \frac{1}{2} \right)^2 \cdot \frac{1}{2} = \frac{1}{8} \]

YES: \[ P(A \cap C) = P(A) \cdot P(C) \]
\[ A \cap C = \{ RRRRR \} \implies P(A \cap C) = \frac{1}{32} \]
\[ P(A) \cdot P(C) = \frac{1}{2} \cdot \frac{1}{16} = \frac{1}{32} \]

NO: \[ P(B \cap C) \neq P(B) \cdot P(C) \]
\[ B \cap C = \{ SSSSS \} \implies P(B \cap C) = \frac{1}{32} \]
\[ P(B) \cdot P(C) = \frac{1}{4} \cdot \frac{1}{16} = \frac{1}{64} \]