Quiz 6 Solutions

Problem 1 A weighted coin with \( p(H) = \frac{2}{3} \) is tossed 20 times. Determine the probability \( p \) that the numbers of heads occurring is between 12 and 15 by

i) Using the binomial distribution

Solution

\[
\begin{align*}
b(12;20,2/3) &= \binom{20}{12} \left( \frac{2}{3} \right)^{12} \left( \frac{1}{3} \right)^8 \\
b(13;20,2/3) &= \binom{20}{13} \left( \frac{2}{3} \right)^{12} \left( \frac{1}{3} \right)^8 \\
b(14;20,2/3) &= \binom{20}{14} \left( \frac{2}{3} \right)^{12} \left( \frac{1}{3} \right)^8 \\
b(15;20,2/3) &= \binom{20}{15} \left( \frac{2}{3} \right)^{12} \left( \frac{1}{3} \right)^8 \\
p &= b(12;20,2/3) + b(13;20,2/3) + b(14;20,2/3) + b(15;20,2/3)
\end{align*}
\]

ii) Using the normal approximation to the binomial distribution

Solution

\[
\begin{align*}
\mu &= np = 20 \cdot \left( \frac{2}{3} \right) = 13.3 \\
\tau &= \sqrt{npq} = \sqrt{20 \cdot \frac{1}{3} \cdot \frac{2}{3}} = 2.1
\end{align*}
\]

\[
p \approx p(11.5 \leq X \leq 15.5) \\
\begin{align*}
&= p\left( \frac{11.5 - 13.3}{2.1} \leq X^* \leq \frac{15.5 - 13.3}{2.1} \right) \\
&= p\left( -0.86 \leq X^* \leq 1.05 \right) \\
&= p\left( -0.86 \leq X^* \leq 0 \right) + p\left( 0 \leq X^* \leq 1.05 \right) \\
&= p\left( 0 \leq X^* \leq 0.86 \right) + p\left( 0 \leq X^* \leq 1.05 \right) \\
&= 0.3051 + 0.3531 = 0.6582
\end{align*}
\]
i) What does this problem imply about the normal approximation.

Solution

Since \( p \) and \( q \) are both away from zero, and \( n \) is large enough (in comparison) the normal distribution is a good approximation of the binomial distribution.

Problem 2 170 students take an exam. Suppose the scores are normally distributed with mean 62 and standard deviation 4 points. If 90 is an A, 80 is a B ... and 49 and below is a failing F, determine how many people get A's. Also determine how many people fail.

Solution

\[
p(X \geq 90) = P(X^* \geq \frac{90 - 62}{4}) = P(X^* \geq 7) = 0
\]

No students gets an A. Similarly

\[
p(X \leq 49) = P(X^* \leq \frac{49 - 62}{4}) = P(X^* \leq -3.25) = 0.5 - P(0 \leq X^* \leq 3.25) = 0.0006
\]

Thus, the number of people who fail is \( 170 \cdot 0.0006 = 0.1 \). No students gets a failing grade.

Problem 3 Suppose that there are 250 misprints distributed randomly throughout a book that is 225 pages. Find the probability that:

1) a given page has no misprint.

Solution

Your text propose in that case that a poisson approximation be used with \( n = 500 \), and \( p = 1/225 \) so

\( \lambda = \frac{500}{225} = 1.1 \)

\[
p(0; 1.1) = \frac{(1.1)^0 e^{-1.1}}{0!} = e^{-1.1} \approx 0.333
\]

2) a given page has 2 misprints.

Solution

\[
p(2; 1.1) = \frac{(1.1)^2 e^{-1.1}}{2!} \approx 0.201
\]