

**Quiz 5 Solutions**

**Problem 1** Two teams are playing in the 2nd round of the playoffs. Assume the rules have changed such that the first team to win 2 games in a row or 4 games total advances to the next round. If the probability that team A wins any single game is 1/3, find the expected number of games the teams will play in this round.

**Solution** Let $X$ be a random variable that assigns the number of games. Let $L$ be "team A looses and W team A wins."

$$P(X = 2) = P(WW) + P(LL) = \left( \frac{1}{3} \right)^2 + \left( \frac{2}{3} \right)^2 = \frac{5}{9}$$

$$P(X = 3) = P(WLL) + P(LWW) = \left( \frac{1}{3} \right) \left( \frac{2}{3} \right)^2 + \left( \frac{2}{3} \right) \left( \frac{1}{3} \right)^2 = \frac{6}{27}$$

$$P(X = 4) = P(LWLL) + P(WLWW) = \left( \frac{1}{3} \right)^3 \left( \frac{2}{3} \right) + \left( \frac{2}{3} \right)^3 \left( \frac{1}{3} \right) = \frac{10}{27}$$

$$P(X = 5) = P(WLWL) + P(WLWLW) = \left( \frac{1}{3} \right)^2 \left( \frac{2}{3} \right)^3 + \left( \frac{2}{3} \right)^2 \left( \frac{1}{3} \right)^3 = \frac{12}{27}$$

$$P(X = 6) = P(WLWLWW) + P(WWLWLL) = \left( \frac{1}{3} \right)^4 \left( \frac{2}{3} \right)^2 + \left( \frac{2}{3} \right)^4 \left( \frac{1}{3} \right)^2 = \frac{20}{27}$$

$$P(X = 7) = P(WLWLWLW) + P(WLWLWLW) + P(WLWLWLW) + P(WLWLWLW)$$

$$= 2 \left( \frac{1}{3} \right)^4 \left( \frac{2}{3} \right)^3 + 2 \left( \frac{2}{3} \right)^4 \left( \frac{1}{3} \right)^3 = \frac{48}{27}$$

So

$$E(X) = 2 \left( \frac{5}{3^2} \right) + 3 \left( \frac{6}{3^3} \right) + 4 \left( \frac{10}{3^4} \right) + 5 \left( \frac{12}{3^5} \right) + 6 \left( \frac{20}{3^6} \right) + 7 \left( \frac{48}{3^7} \right) = \frac{2068}{3^6}$$

Therefore they expect to play around 3 games.

**Problem 2** Consider the following joint distribution table:

<table>
<thead>
<tr>
<th>X \ Y</th>
<th>-3</th>
<th>-1</th>
<th>2</th>
<th>3</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.1</td>
<td>.3</td>
<td>0</td>
<td>.1</td>
<td>.5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>.1</td>
<td>.2</td>
<td>.2</td>
<td>.5</td>
</tr>
<tr>
<td>Sum</td>
<td>.1</td>
<td>.4</td>
<td>.2</td>
<td>.3</td>
<td>1</td>
</tr>
</tbody>
</table>


1) Find $E(X)$ and $E(Y)$.

Solution

$$E(X) = 1 \cdot .5 + 2 \cdot .5 = \frac{3}{2}$$

$$E(Y) = -3 \cdot .1 - 1 \cdot .4 + 2 \cdot .2 + 3 \cdot .3 = \frac{3}{5}$$

2) Find the Cov($X,Y$).

Solution

$$E(XY) = (-3)(1)(.1) + (-1)(1)(.3) + (3)(1)(.1) + (2)(-1)(.1) + (2)(2)(.2) + (3)(2)(.2) = 1.5$$

So

$$Cov(X,Y) = E(XY) - E(X)E(Y) = 1.5 - (1.5)(.6) = .6$$

3) Find the standard deviation of $X$ and $Y$ and $\rho(X,Y)$.

Solution

$$E(X^2) = (1)^2(.5) + (2)^2(.5) = \frac{5}{2}$$

$$Var(X) = \frac{5}{2} - \left(\frac{3}{2}\right)^2 = \frac{1}{4}$$

$$\sigma_x = \sqrt{Var(X)} = \frac{1}{2}$$

$$E(Y^2) = (-3)^2(.1) + (-1)^2(.4) + (2)^2(.2) + (3)^2(.3) = \frac{25}{4}$$

$$Var(Y) = \frac{25}{4} - \left(\frac{3}{5}\right)^2 = \frac{111}{25}$$

$$\sigma_y = \sqrt{Var(Y)} = \frac{\sqrt{111}}{5}$$

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sigma_x \sigma_y} = \frac{1.5}{\frac{\sqrt{111}}{5}} = \frac{75}{\sqrt{111}}$$

4) Are $X$ and $Y$ independent?

Solution

No, because $Cov(X,Y) \neq 0$. Please note that if you find a $Cov(X,Y) = 0$ for another problem, it does NOT imply that the events are independent.
**Problem 3** Let $X$ be a continuous random variable with distribution

$$f(x) = \begin{cases} 
1/8x & \text{for } 0 \leq x \leq 4 \\
0 & \text{elsewhere}
\end{cases}$$

1) Find the probability that $x \leq 2.5$.

**Solution**

$$\int_0^{2.5} \frac{x}{8} \, dx = \frac{1}{8} \frac{(2.5)^2}{2} = \frac{25}{64}$$

2) Plot the graph of the cumulative distribution function.

**Solution**

$$F(x) = \begin{cases} 
0 & x < 0 \\
\frac{x^2}{16} & \text{for } 0 \leq x \leq 4 \\
1 & x > 4
\end{cases}$$