Quiz 3 Solutions

Problem 1 Let $P$ be a probability function on the set $S = a, b, c, d$. Find the probability of $P(a)$ if:

1) $P(b) = 1/4, P(c) = 1/8, P(d) = 1/6.$

Solution By definition a probability function must =1. So we must have:

$$P(a) + P(b) + P(c) + P(d) = 1$$

$$P(a) = 1 - (P(b) + P(c) + P(d))$$

$$P(a) = 1 - \left(\frac{1}{4} + \frac{1}{8} + \frac{1}{6}\right)$$

$$P(a) = 1 - \left(\frac{13}{24}\right)$$

$$P(a) = \frac{11}{24}$$

2) $P(b) = 1/2, P(c) = 1/4, P(c) = 2P(d).$

Solution Similarly

$$P(a) + P(b) + P(c) + P(d) = 1$$

$$P(a) = 1 - (P(b) + P(c) + P(d))$$

$$P(a) = 1 - (P(b) + P(c) + P(c)/2)$$

$$P(a) = 1 - \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)$$

$$P(a) = 1 - \left(\frac{7}{8}\right)$$

$$P(a) = \frac{1}{8}$$

Problem 2 A man is dealt 3 cards from a deck of 52. Find the probability that:

1) 2 are spades and one is diamond.

Solution We first need to look for our sample space which is $\binom{52}{3}$. Then there are 13 spades in a deck and we want to choose 2 from them. There are 13 diamonds from which we want to choose 1 of them. So

$$P(2 \text{ spade and 1 diamond}) = \frac{\binom{13}{2} \binom{13}{1}}{\binom{52}{3}}$$
2) All three are diamond

Solution

\[ P(\text{3 diamond}) = \left( \frac{13}{32} \right) \]

Problem 3 The two points \( a_1 \) and \( a_2 \) are selected at random from the real line such that \(-1 \leq a_1 \leq 1\) and \(-2 \leq a_2 \leq 0\). Find the probability that the distance between \( a_1 \) and \( a_2 \) is less than 1.

Solution Recall that the distance between \( a_1 \) and \( a_2 \) is given by \( d = |a_2 - a_1|\). In other words, \(-1 \leq a_2 - a_1 \leq 1\) or \( a_2 \leq a_1 + 1 \) and \( a_2 \geq a_1 - 1 \). So all points inside the gray area (in the picture below) have a distance less than 1.

\[
P(d \leq 1) = \frac{\text{Area}(S)}{\text{Total Area}}
\]

\[
= \frac{2}{4}
\]

\[
= \frac{1}{2}
\]