Introduction

The purpose of this assignment is to practice CMOS logic design style and Boolean algebra. For CMOS logic design style, we implement a Boolean function and show the design with a schematic diagram. For Boolean algebra, first the exercise is to help you practice the application of Boolean algebra theorems to transform and reduce Boolean expressions. The second goal is to help you learn how to go from the world of Boolean expressions to the world of digital circuits. The final goal is to help you translate a problem described in Boolean algebraic expression to digital logic. We hope you can think of why each of these exercises is useful when designing digital circuits.

1 Implementation of Switching Function using CMOS Logic

Given a circuit with three binary inputs \((a, b, c)\) and a binary output function \(y(a, b, c) = (ab + bc)'\), implement the function using CMOS logic technology. Draw the design with a schematic diagram.

2 Application of Boolean Algebra and Shannon Expansion

2.1 Prove the following using Boolean algebra.

A. \(a + ab = a\).
B. \(a \cdot (a + b) = a\).
C. \((be + d)(a + c + d')(a + be + c) = (be + d)(a + c + d')\)

2.2 Prove the above three equations using Shannon’s expansion.

3 Prove or disprove the following statements in Boolean expressions

A. If \(a = b + c\) and \(a = c\), then \(b = 0\).
B. If \(a = bc\) and \(a = c\), then \(b = 1\).
C. \((b + de)(a + c + d'e')(a + b + c) = (b + de)(a + c + d'e')\)
4 Translate From a Digital Problem to Boolean Expressions

A circuit has four binary inputs \((a_3, a_2, a_1, a_0)\) and one binary output \(y\). Output \(y\) is true when the following arithmetic operation satisfies the inequality: \(4 \times a_3 + 3 \times a_2 + 2 \times a_1 + a_0 \geq 4\).
A. Write the truth table.
B. Write the function in sum-of-products canonical form.
C. Write the function in product-of-sums canonical form.

5 Boolean Algebra and Implementation

A. Simplify each of the following two Boolean equations (using Boolean algebra, in particular consensus theorem). Write the results in Boolean expressions.
B. Convert the simplified expressions with logic using \(AND\), \(OR\) gates with bubbles for (True, False) inversion. Draw the logic diagram.
C. List the numbers of literals and operators versus the numbers of gates, nets, and pins in the schematic diagrams
   i. \(y(a, b, c, d, e) = a + a'b + a'b'c + a'b'c'd + a'b'c'd'e\)
   ii. \(y(a, b, c, d) = abc + a'b(cd + cd') + b'(ad + c'd') + d(a'b' + c'b)\)

Assignment Turn-in

Once you’re finished with the assignment, turn it in as a pdf on Gradescope.