Sampling and Aliasing, and The Discrete Fourier Transform

Image Processing

CSE 166

Lecture 6

Announcements

- Assignment 2 is due today, 11:59 PM
- Assignment 3 will be released Apr 20
- Reading
  - Chapter 4: Filtering in the Frequency Domain

Overview: Image processing in the frequency domain

Image in spatial domain $f(x,y)$ → **Fourier transform** → Image in frequency domain $F(u,v)$

↓

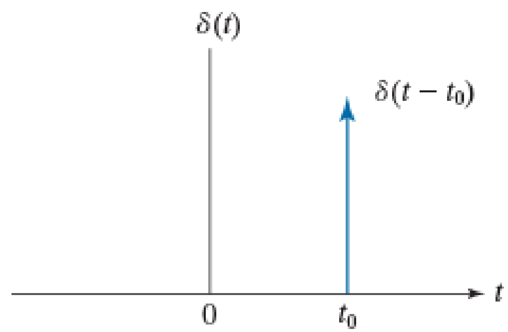
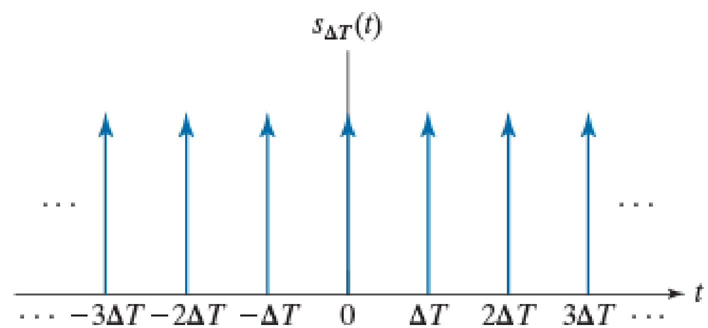
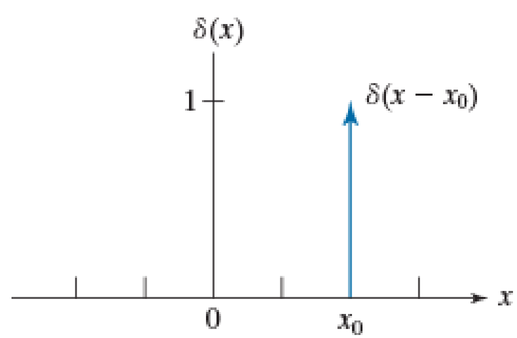
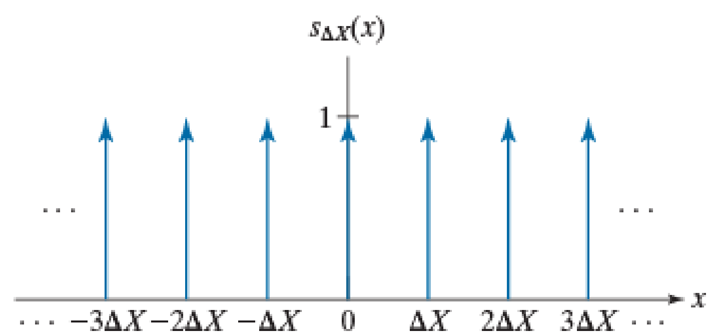
Frequency domain processing

↓

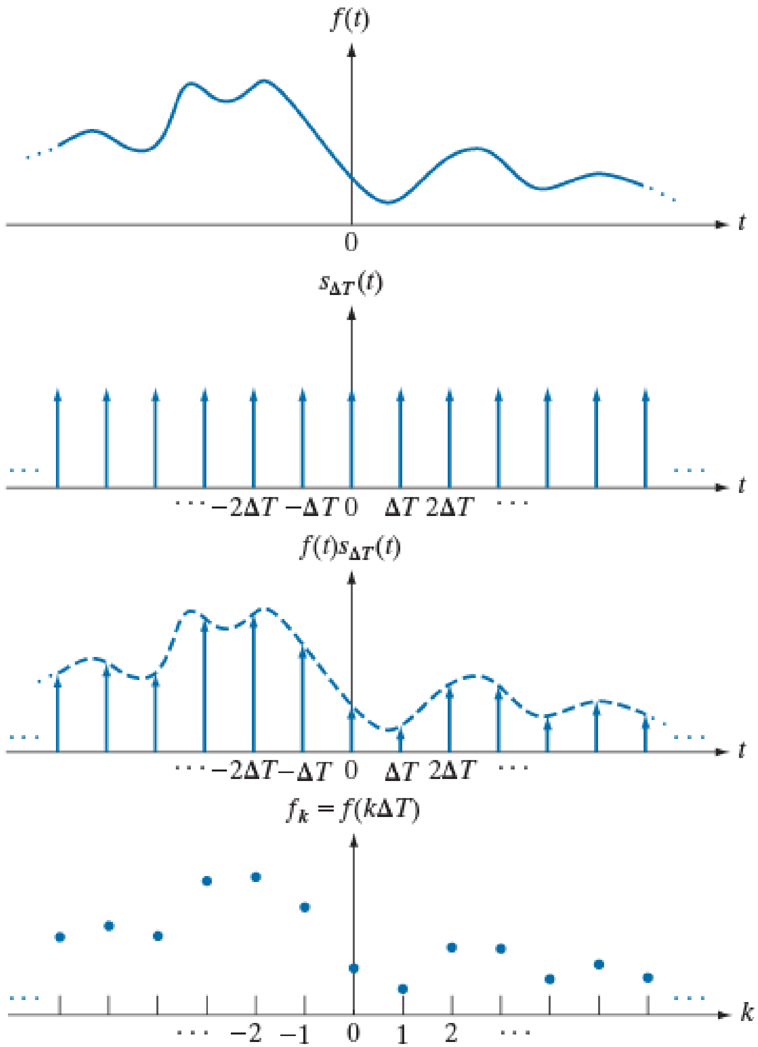
Image in frequency domain $G(u,v)$ → **Inverse Fourier transform** → Image in spatial domain $g(x,y)$

Jean-Baptiste Joseph Fourier
1768-1830

1D impulse function and impulse train

	Impulse function	Impulse train
Continuous		
Discrete		

Sampling



a
b
c
d

FIGURE 4.5
(a) A continuous function. (b) Train of impulses used to model sampling. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of impulses. (The dashed line in (c) is shown for reference. It is not part of the data.)

Sampling

- Fourier transform of sampled function

$$\tilde{F}(\mu) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F\Big(\mu - \frac{n}{\Delta t}\Big)$$

is an infinite, periodic sequence of copies of $F(\mu)$

Sampling

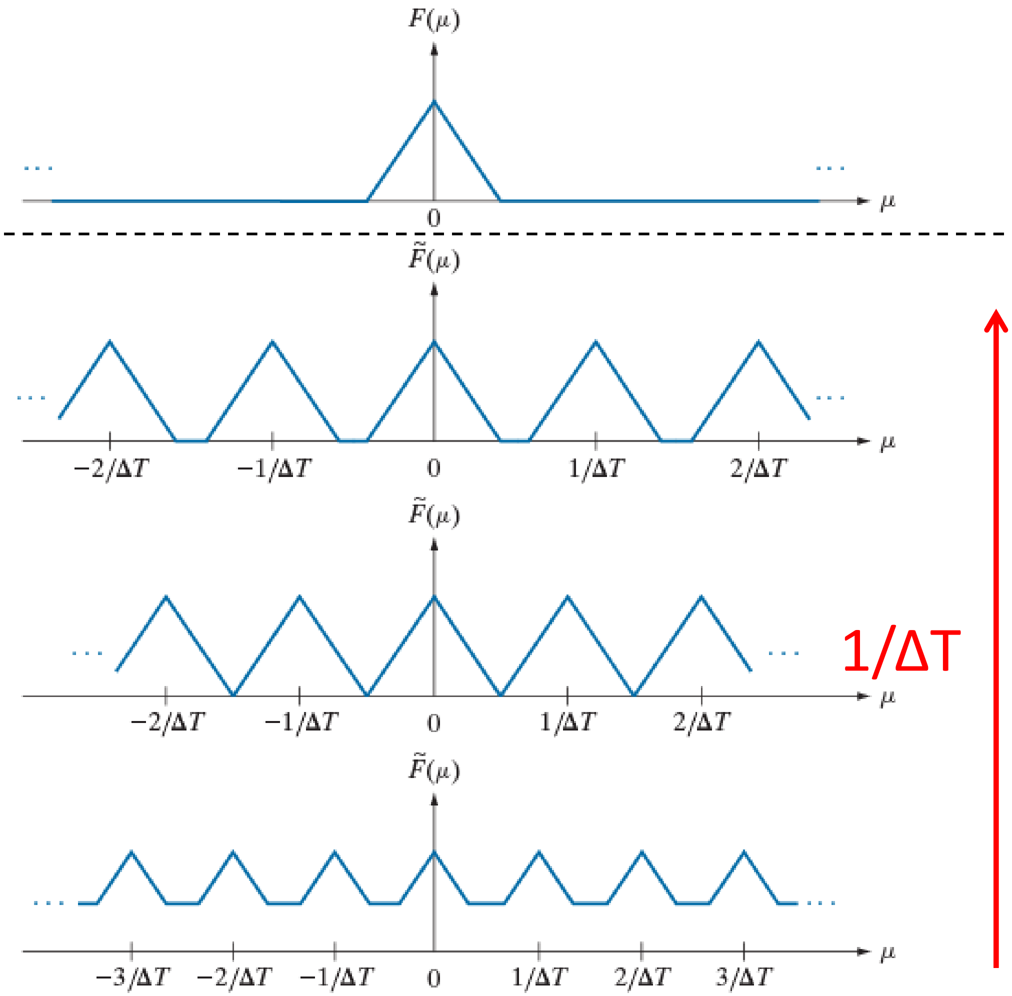
Fourier transform of function

Fourier transforms of **sampled** function

Over-sampled

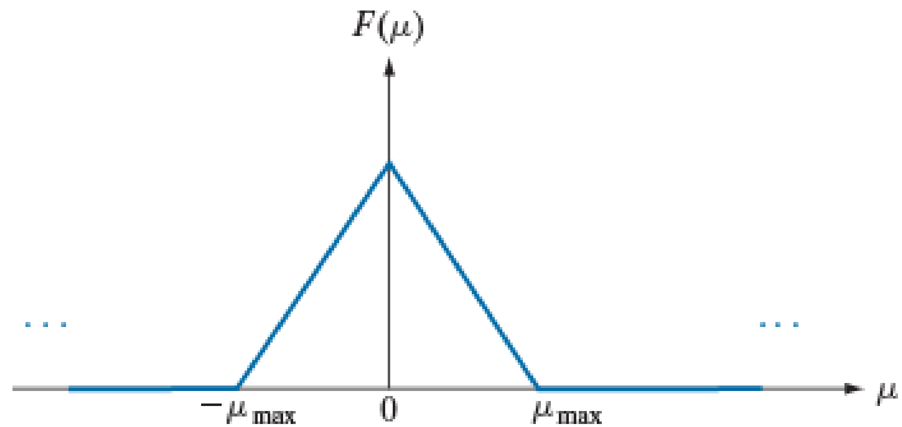
Critically-sampled

Under-sampled



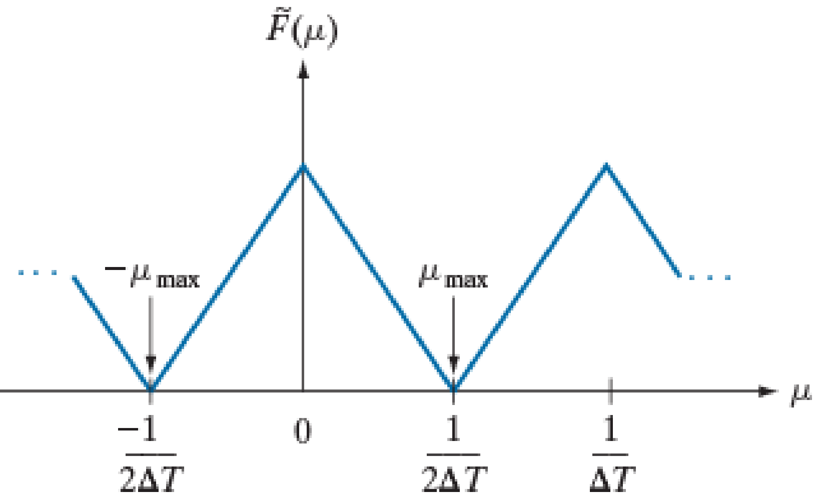
The sampling theorem

Fourier transform of function



Fourier transform of **sampled** function

Critically-sampled



Recovering F(μ) from F̃(μ)

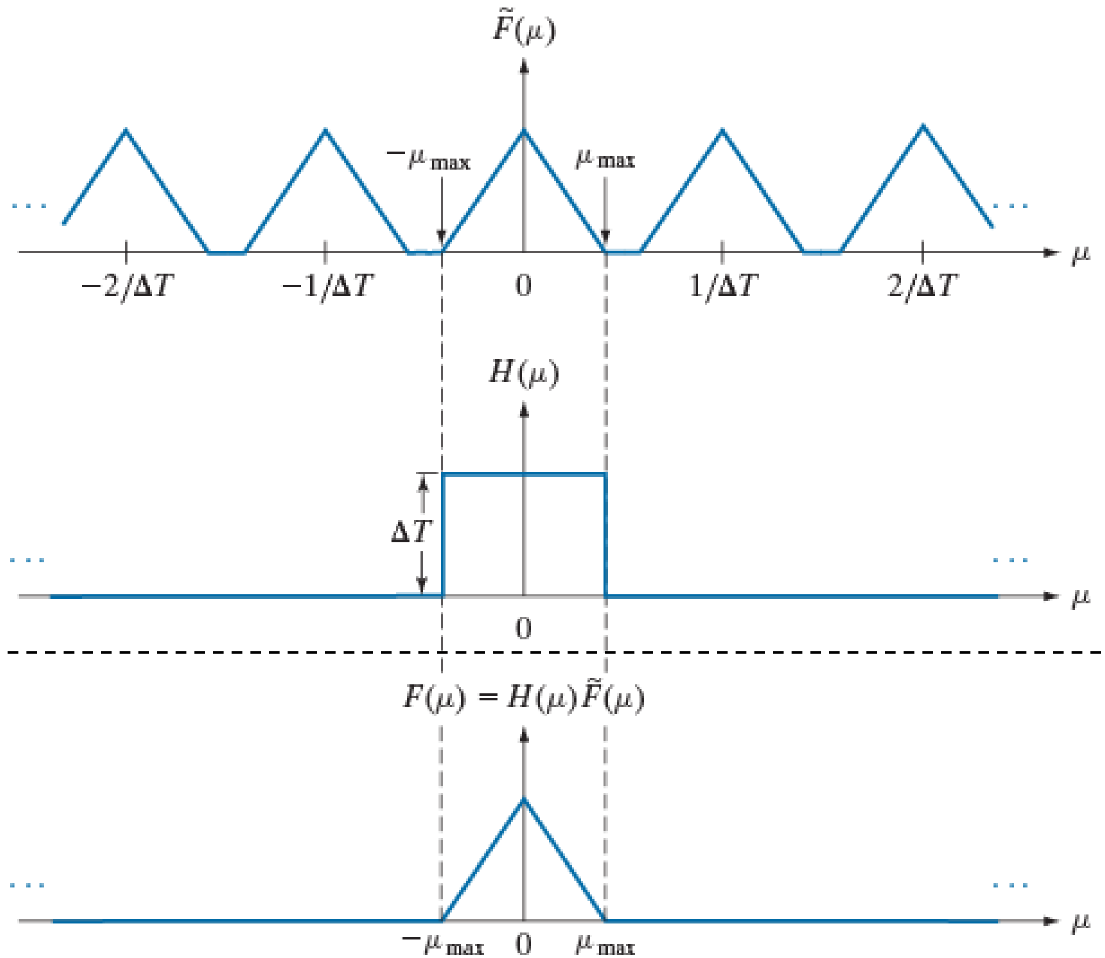
Fourier transform of **sampled** function

Over-sampled

Ideal lowpass filter

Product of above

Recovered



Aliasing

Continuous　　　　Sampled

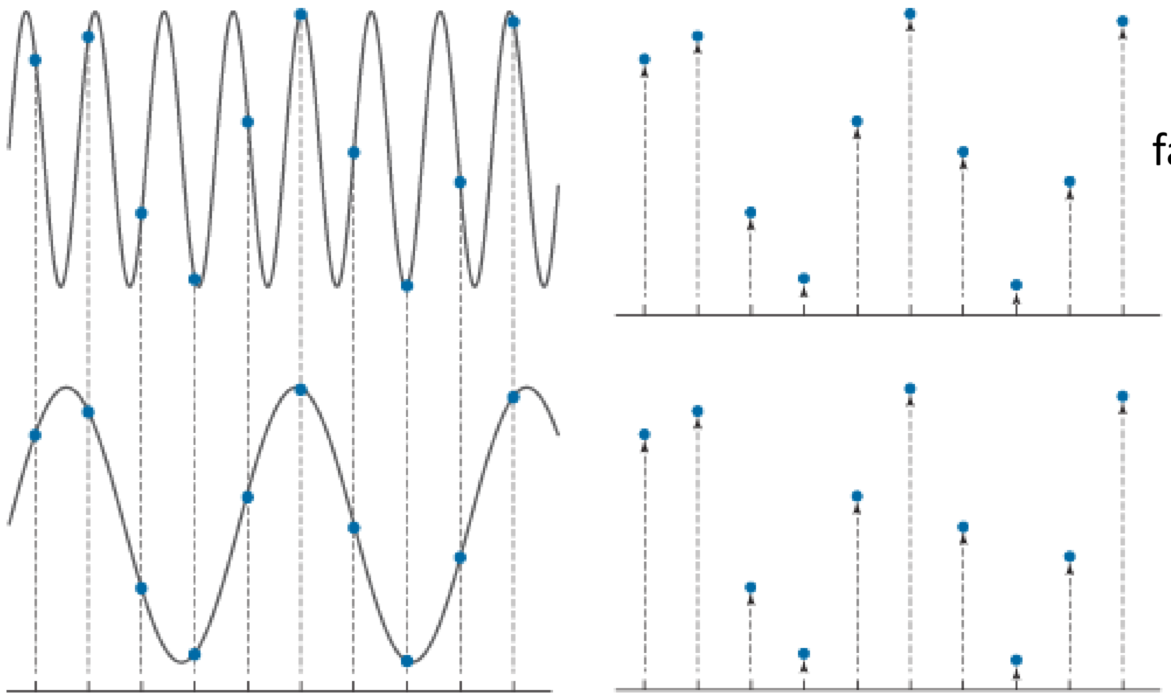
Under-sampled

Alias: a false identity

Different

Identical

Over-sampled



Sampled at same rate

Aliasing

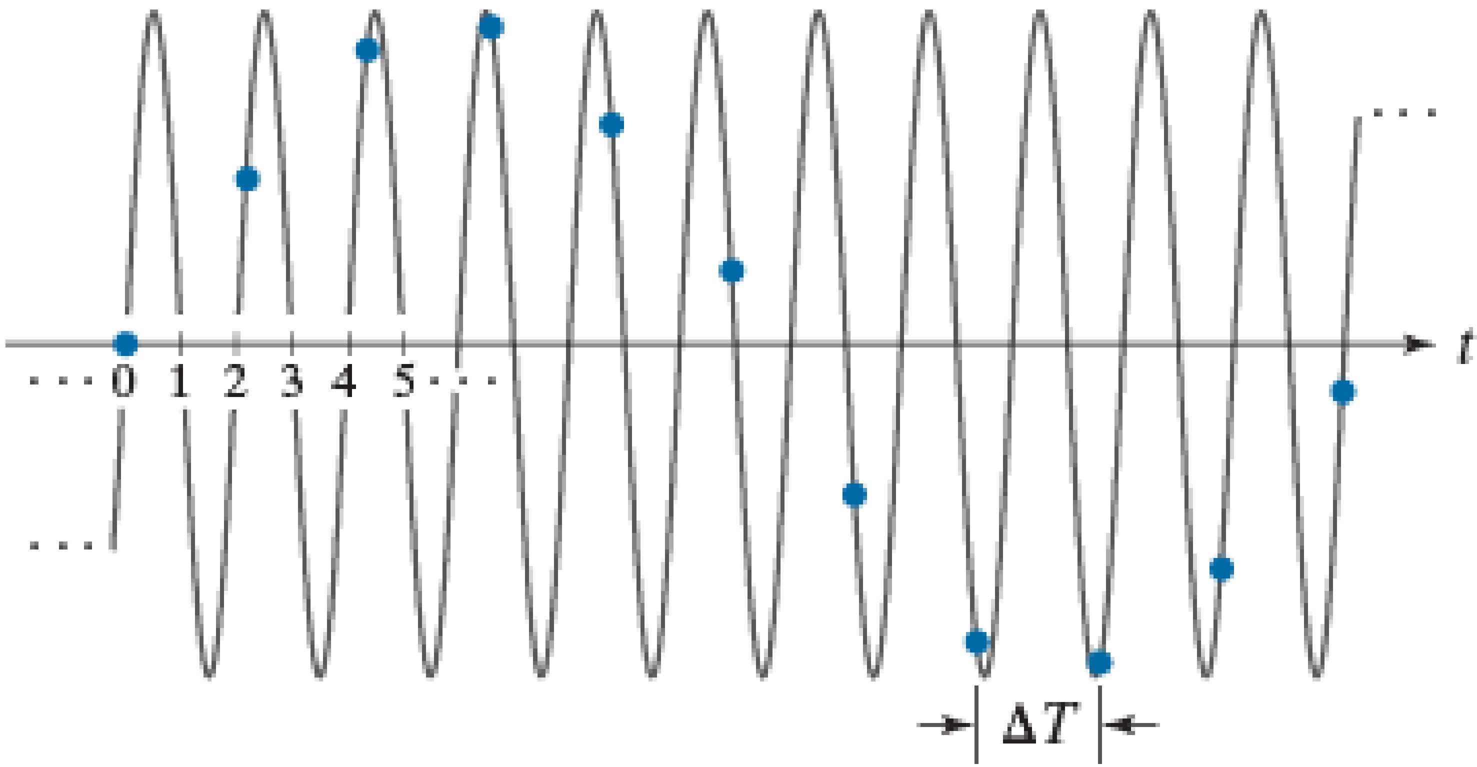


FIGURE 4.11 Illustration of aliasing. The under-sampled function (dots) looks like a sine wave having a frequency much lower than the frequency of the continuous signal. The period of the sine wave is 2 s, so the zero crossings of the horizontal axis occur every second. ΔT is the separation between samples.

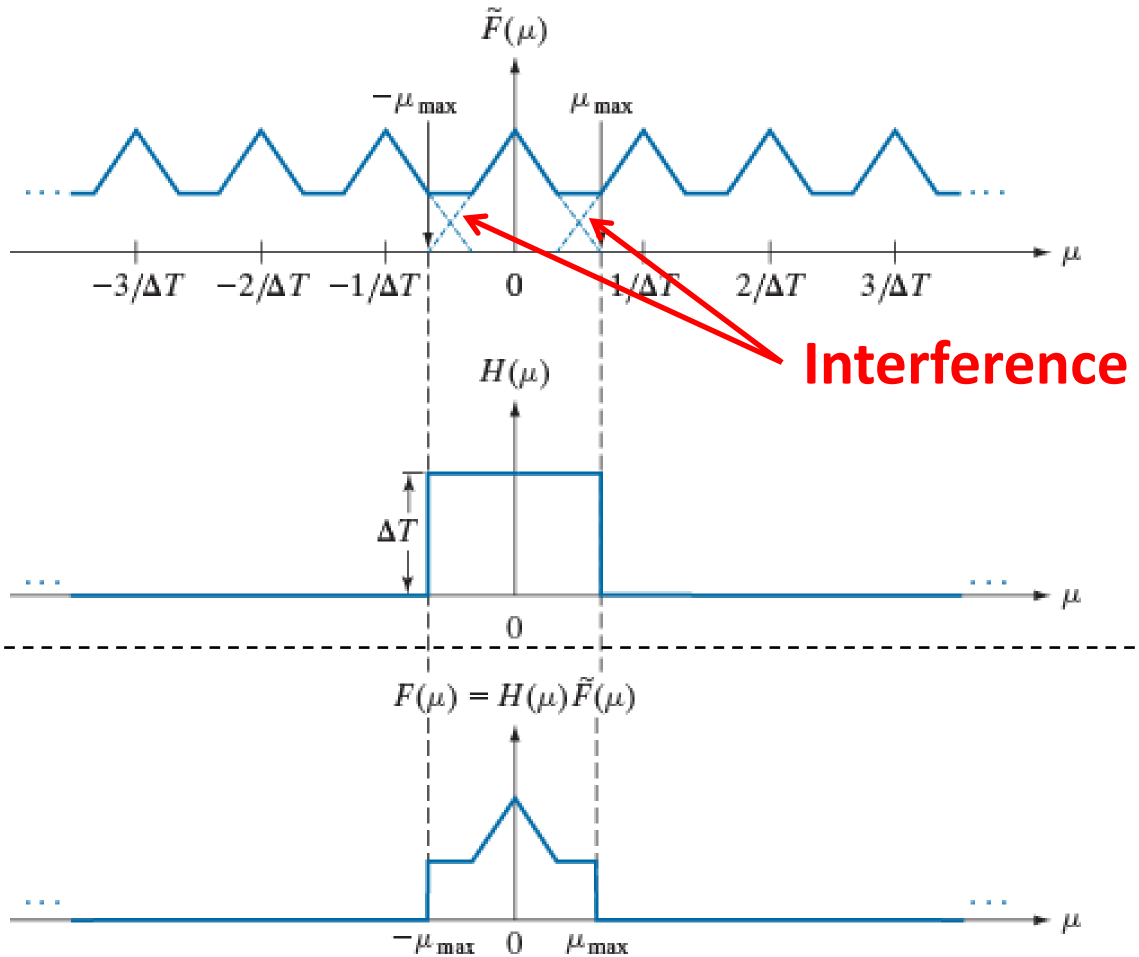
Aliasing

Fourier transform of **under-sampled** function

Ideal lowpass filter

Product of above

Imperfect recovery



1D discrete Fourier transform (DFT)

- (Forward) Fourier transform

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi ux/M} \quad u = 0, 1, 2, \ldots, M-1$$

- Inverse Fourier transform

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{j2\pi ux/M} \quad x = 0, 1, 2, \ldots, M-1$$

Next Lecture

- Filtering in the frequency domain
- Reading
  - Chapter 4: Filtering in the Frequency Domain