The Continuous Fourier Transform

Image Processing
CSE 166
Lecture 5
Announcements

• Assignment 2 is due Apr 15, 11:59 PM
• Assignment 3 will be released Apr 20
• Reading
  – Chapter 4: Filtering in the Frequency Domain
Overview: Image processing in the frequency domain

Image in spatial domain $f(x,y)$ → Fourier transform → Image in frequency domain $F(u,v)$ → Frequency domain processing → Image in frequency domain $G(u,v)$ → Inverse Fourier transform → Image in spatial domain $g(x,y)$

Jean-Baptiste Joseph Fourier
1768-1830
Review

• Complex numbers \( C = R + jI \)
• Euler’s formula \( e^{j\theta} = \cos \theta + j \sin \theta \)
• Complex functions
1D Fourier series

Sines and cosines

Periodic function

Weighted by magnitude

Shifted by phase

Period $T$
1D Fourier transform

Sines and cosines

Periodic function

Period $T$  Frequency $\mu$

$\mu_1$

$\mu_2$

$\mu_3$

$\mu_4$

Magnitude

Frequency

CSE 166, Spring 2020
1D continuous Fourier transform

• (Forward) Fourier transform

\[ \mathcal{F} \{ f(t) \} = F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi \mu t} \, dt \]

• Inverse Fourier transform

\[ \mathcal{F}^{-1} \{ F(\mu) \} = f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi \mu t} \, d\mu \]
1D continuous Fourier transform

- Example: box function

**Figure 4.4** (a) A box function, (b) its Fourier transform, and (c) its spectrum. All functions extend to infinity in both directions. Note the inverse relationship between the width, $W$, of the function and the zeros of the transform.
Convolution theorem

• Fourier transform of 1D continuous convolution
  \[ \mathcal{F} \{ f(t) \star h(t) \} = H(\mu)F(\mu) \]

• Convolution theorem
  \[ f(t) \star h(t) \iff H(\mu)F(\mu) \]
  \[ f(t)h(t) \iff H(\mu)\star F(\mu) \]
Next Lecture

• Sampling and aliasing, and the discrete Fourier transform
• Reading
  – Chapter 4: Filtering in the Frequency Domain