

Spatial Filtering

Image Processing

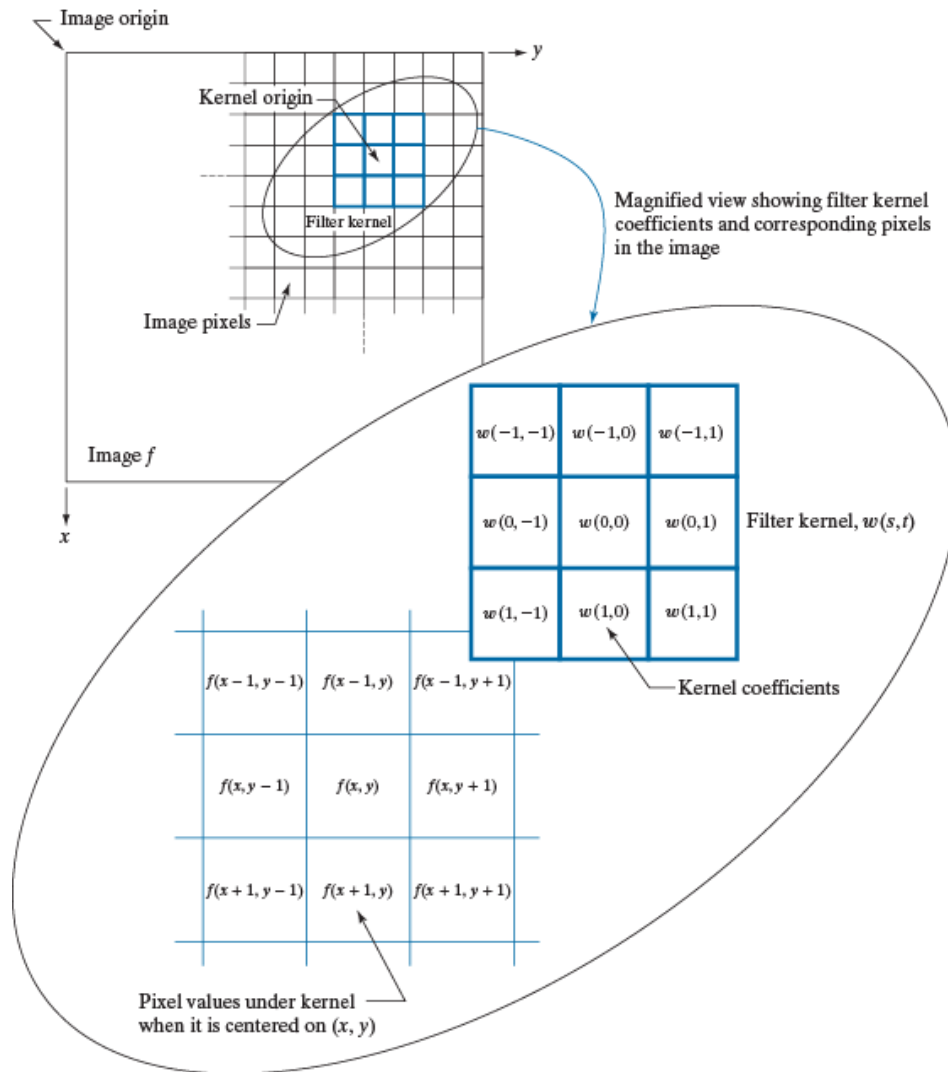
CSE 166

Lecture 4

Announcements

- Assignment 1 is due today, 11:59 PM
- Assignment 2 will be released today
 - Due Apr 15, 11:59 PM
- Reading
 - Chapter 3: Intensity Transformations and Spatial Filtering

Spatial filtering (2D)



2D correlation

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

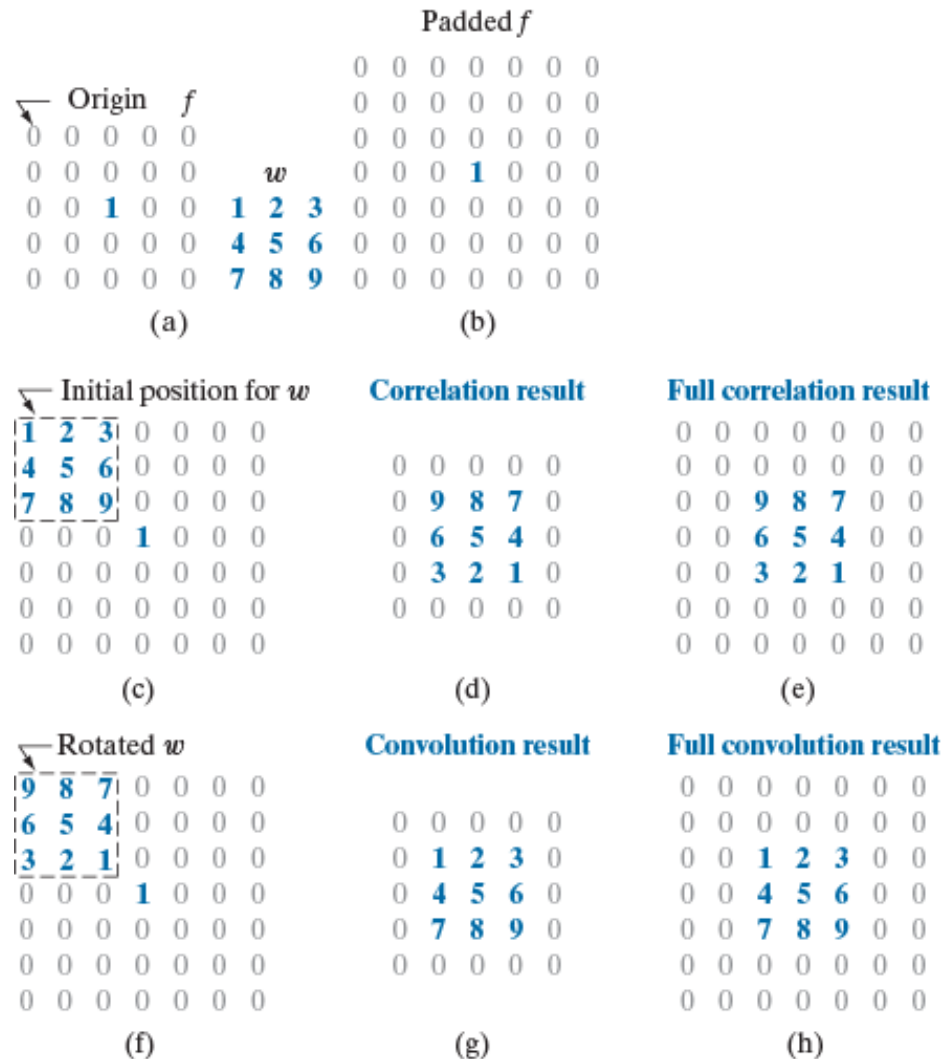
2D convolution

$$w(x, y) \blackstar f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x - s, y - t)$$

Correlation and convolution (2D)

FIGURE 3.36

Correlation (middle row) and convolution (last row) of a 2-D kernel with an image consisting of a discrete unit impulse. The 0's are shown in gray to simplify visual analysis. Note that correlation and convolution are functions of x and y . As these variable change, they *displace* one function with respect to the other. See the discussion of Eqs. (3-45) and (3-46) regarding full correlation and convolution.



Correlation and convolution

- Convolution is commutative and associative, correlation is not

TABLE 3.5

Some fundamental properties of convolution and correlation. A dash means that the property does not hold.

Property	Convolution	Correlation
Commutative	$f \star g = g \star f$	—
Associative	$f \star (g \star h) = (f \star g) \star h$	—
Distributive	$f \star (g + h) = (f \star g) + (f \star h)$	$f \star (g + h) = (f \star g) + (f \star h)$

Smoothing kernels

$$\frac{1}{9} \times$$

1	1	1
1	1	1
1	1	1

Average
(box kernel)

$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

Weighted average
(Gaussian kernel)

Smoothing with box kernel

Input
image



3x3



11x11



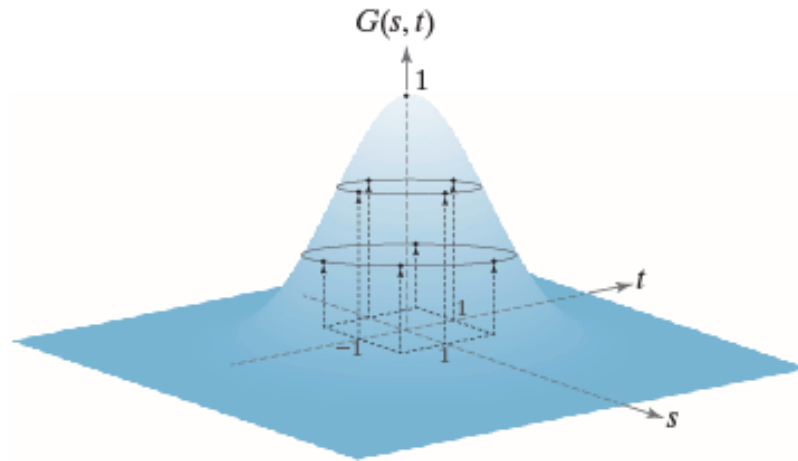
21x21

Smoothing with Gaussian kernel

a b

FIGURE 3.41

(a) Sampling a Gaussian function to obtain a discrete Gaussian kernel. The values shown are for $K = 1$ and $\sigma = 1$. (b) Resulting 3×3 kernel [this is the same as Fig. 3.37(b)].



$$\frac{1}{4.8976} \times$$

0.3679	0.6065	0.3679
0.6065	1.0000	0.6065
0.3679	0.6065	0.3679

Standard deviation σ	Percent of total volume under surface
1	39.35
2	86.47
3	98.89

Volume under surface greater than 3σ is negligible

Smoothing with Gaussian kernel



$\sigma = 7$
43x43



$\sigma = 7$
85x85



Difference

Smoothing with Gaussian kernel



Input image



$\sigma = 3.5$
21x21



$\sigma = 7$
43x43

Border padding

v	v	v	v	v	v	v	v	v
v	v	v	v	v	v	v	v	v
v	v	1	2	3	4	5	v	v
v	v	6	7	8	9	10	v	v
v	v	11	12	13	14	15	v	v
v	v	16	17	18	19	20	v	v
v	v	v	v	v	v	v	v	v

Zero padding
when $v = 0$

Constant padding

1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
1	1	1	2	3	4	5	5	5
6	6	6	7	8	9	10	10	10
11	11	11	12	13	14	15	15	15
16	16	16	17	18	19	20	20	20
16	16	16	17	18	19	20	20	20

Replicate padding

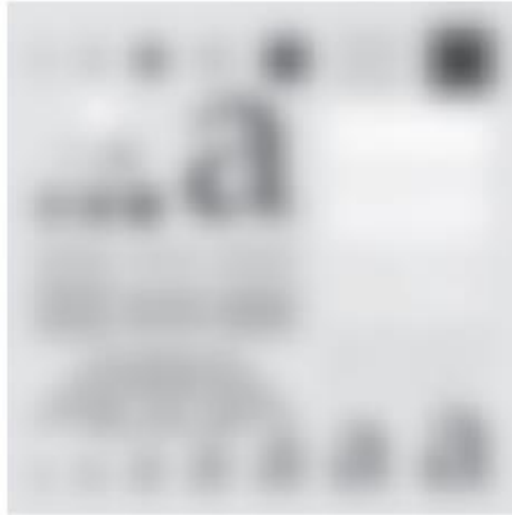
13	12	11	12	13	14	15	14	13
8	7	6	7	8	9	10	9	8
3	2	1	2	3	4	5	4	3
8	7	6	7	8	9	10	9	8
13	12	11	12	13	14	15	14	13
18	17	16	17	18	19	20	19	18
13	12	11	12	13	14	15	14	13

Mirror padding

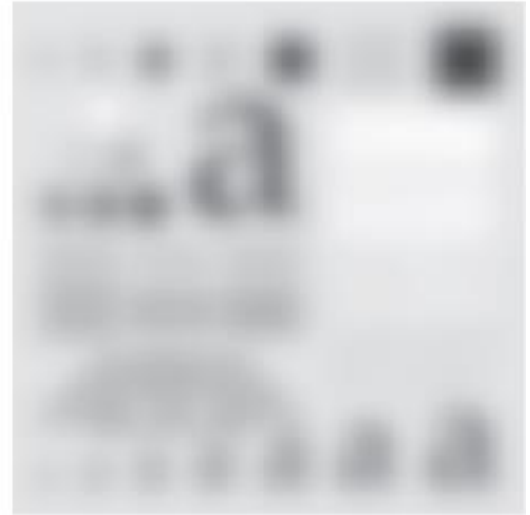
Border padding



Zero
padding

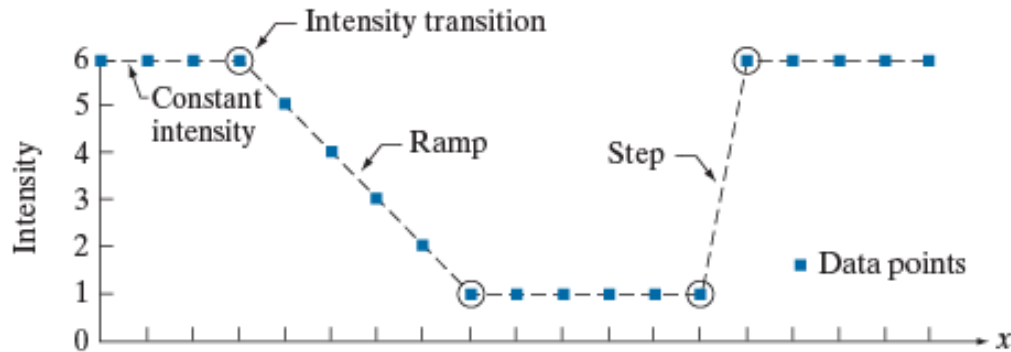


Mirror
padding

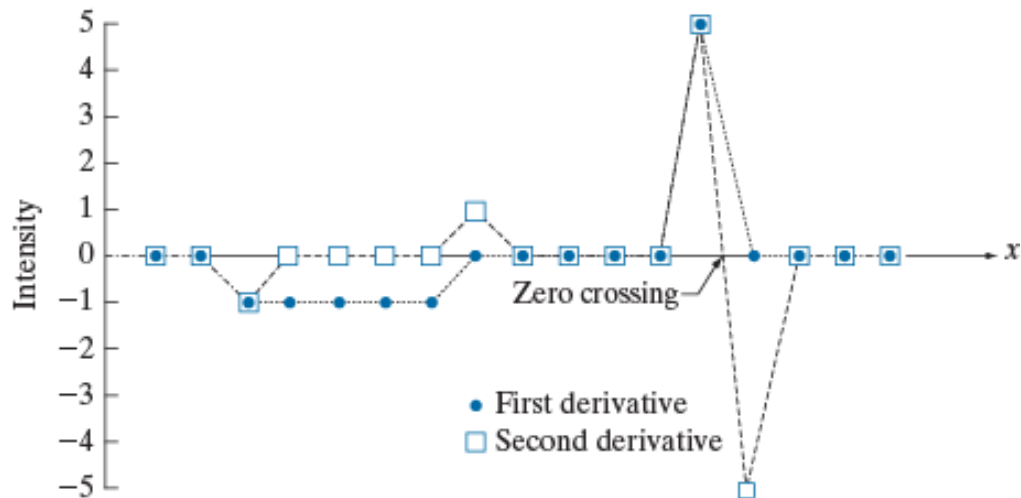


Replicate
padding

Derivatives



Values of scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6	→ x
1st derivative	0	0	0	-1	-1	-1	-1	-1	0	0	0	0	0	0	5	0	0	0	0	
2nd derivative	0	0	0	-1	0	0	0	0	1	0	0	0	0	0	5	-5	0	0	0	



a
b
c

FIGURE 3.50
(a) A section of a horizontal scan line from an image, showing ramp and step edges, as well as constant segments.
(b) Values of the scan line and its derivatives.
(c) Plot of the derivatives, showing a zero crossing. In (a) and (c) points were joined by dashed lines as a visual aid.

Sharpening filters

a b
c d

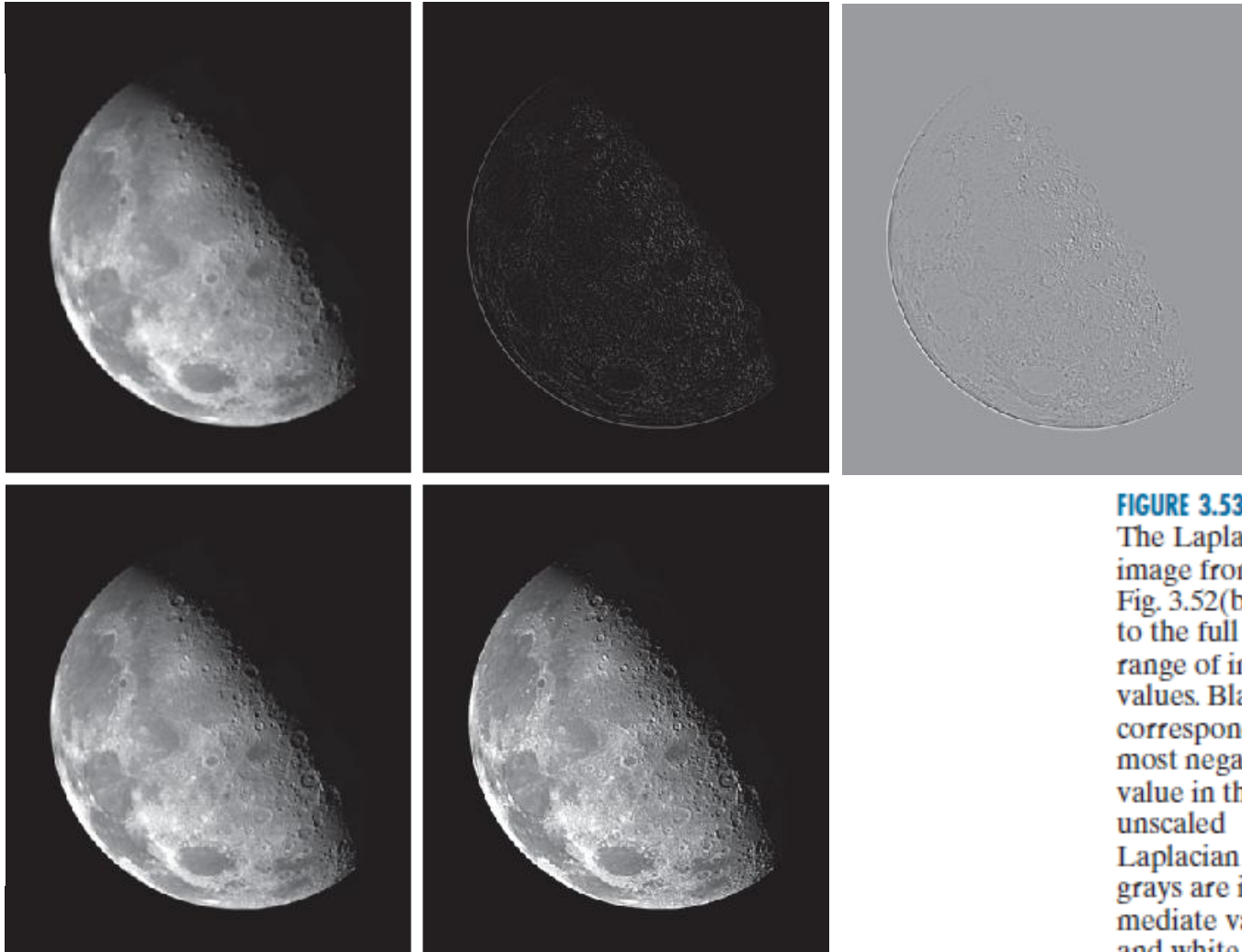
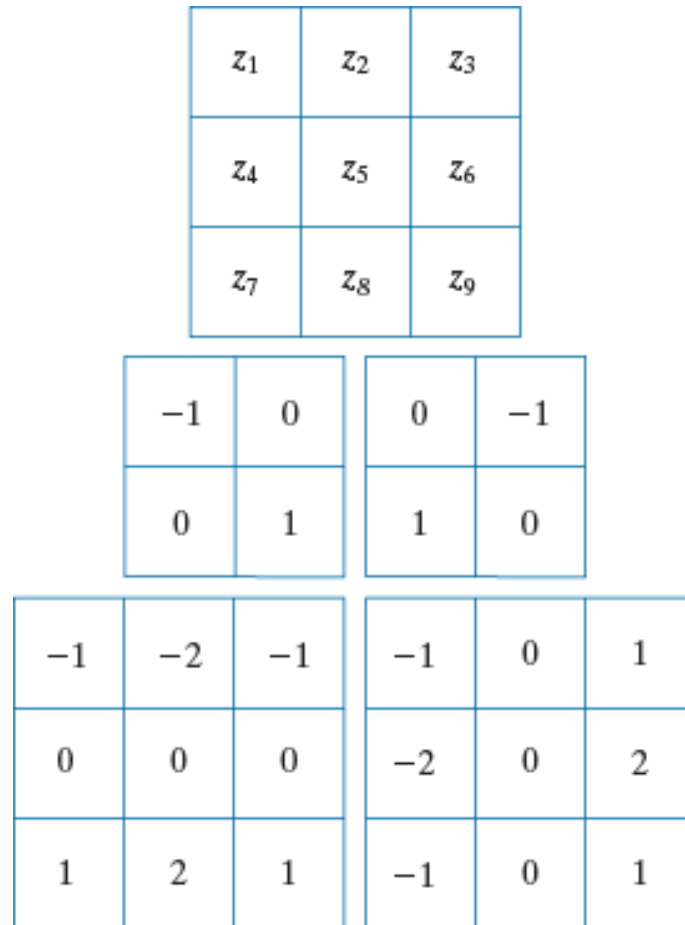


FIGURE 3.52
(a) Blurred image of the North Pole of the moon.
(b) Laplacian image obtained using the kernel in Fig. 3.51(a).
(c) Image sharpened using Eq. (3-63) with $c = -1$.
(d) Image sharpened using the same procedure, but with the kernel in Fig. 3.51(b).
(Original image courtesy of NASA.)

FIGURE 3.53
The Laplacian image from Fig. 3.52(b), scaled to the full $[0, 255]$ range of intensity values. Black pixels correspond to the most negative value in the unscaled Laplacian image, grays are intermediate values, and white pixels corresponds to the highest positive value.

Gradient (first derivatives)

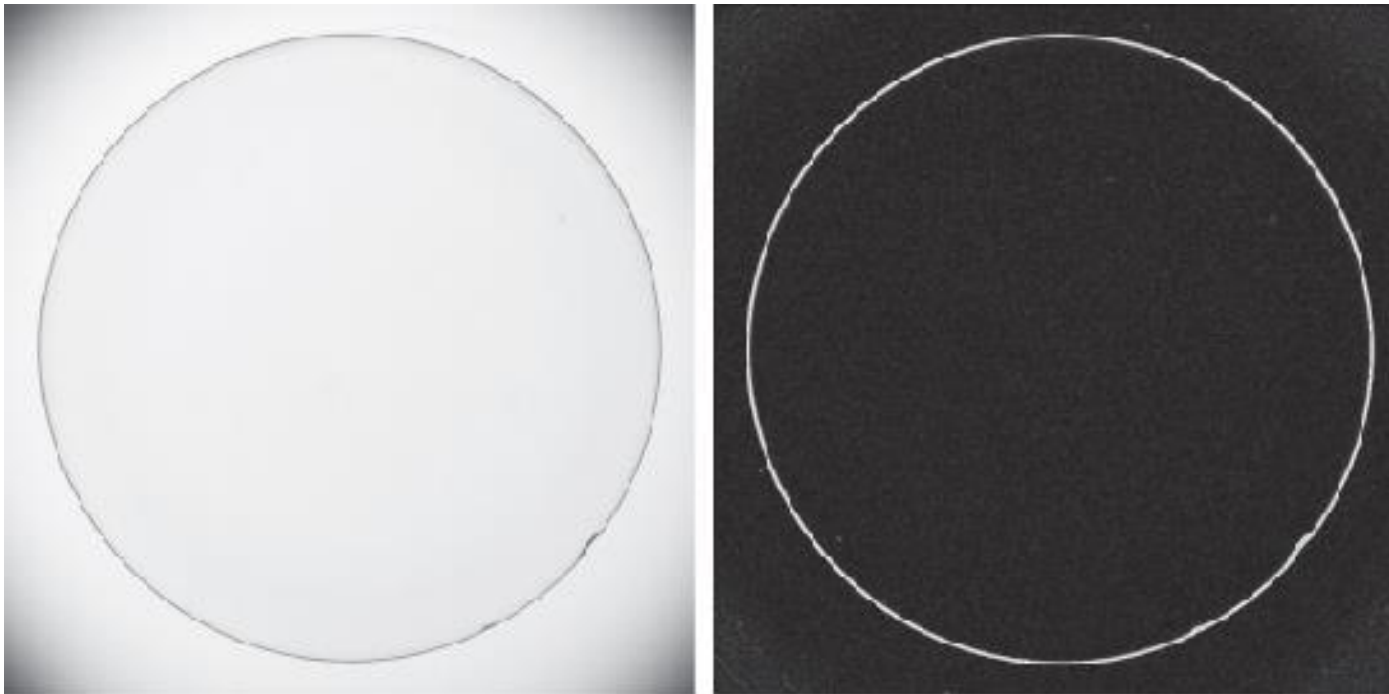


a
b c
d e

FIGURE 3.56

(a) A 3×3 region of an image, where the z s are intensity values. (b)–(c) Roberts cross-gradient operators. (d)–(e) Sobel operators. All the kernel coefficients sum to zero, as expected of a derivative operator.

Magnitude of gradient vector

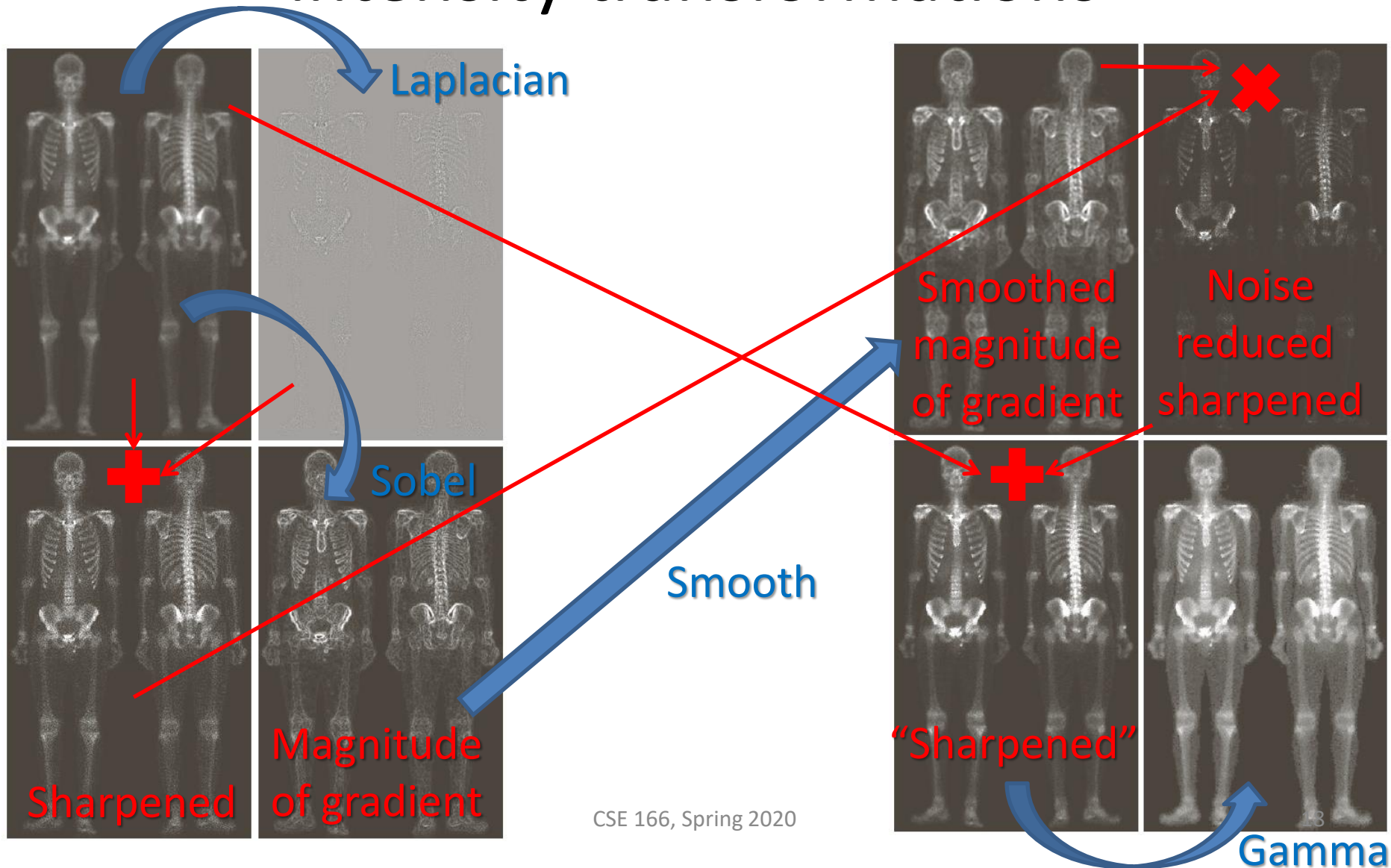


a b

FIGURE 3.57
(a) Image of a contact lens (note defects on the boundary at 4 and 5 o'clock).
(b) Sobel gradient. (Original image courtesy of Perceptics Corporation.)

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Combining spatial filtering and intensity transformations



Next Lecture

- Continuous Fourier transform
- Reading
 - Chapter 4: Filtering in the Frequency Domain