Filtering in the Frequency Domain

Image Processing
CSE 166
Lecture 7
Announcements

• Assignment 3 will be released today
  – Due Apr 29, 11:59 PM

• Reading
  – Chapter 4: Filtering in the Frequency Domain
Overview: Image processing in the frequency domain

Image in spatial domain \( f(x,y) \) \rightarrow \text{Fourier transform} \rightarrow \text{Image in frequency domain} \( F(u,v) \)

Jean-Baptiste Joseph Fourier 1768-1830

Frequency domain processing

Image in frequency domain \( F(u,v) \) \rightarrow \text{Inverse Fourier transform} \rightarrow \text{Image in spatial domain} \( g(x,y) \)

Image in frequency domain \( G(u,v) \) \leftarrow \text{Inverse Fourier transform} \leftarrow \text{Image in spatial domain} \( g(x,y) \)
Continuous Fourier transform

1D

2D
Unit discrete impulse

1D

2D
Impulse train

1D

\[ s_{\Delta x}(x) \]

\[ \cdots -3\Delta X - 2\Delta X - \Delta X 0 \Delta X 2\Delta X 3\Delta X \cdots \]

2D

\[ s_{\Delta T \Delta Z}(t, z) \]

\[ \cdots \Delta Z \Delta T \cdots \]
Fourier transform of sampled function and extracting one period

Over-sampled

Under-sampled

1D

Recovered

2D

Imperfect recovery due to interference
Aliasing

1D

2D

Original

Aliasing
Aliasing in real images

Original  Aliasing  No aliasing

*FIGURE 4.19* Illustration of aliasing on resampled natural images. (a) A digital image of size 772 × 548 pixels with visually negligible aliasing. (b) Result of resizing the image to 33% of its original size by pixel deletion and then restoring it to its original size by pixel replication. Aliasing is clearly visible. (c) Result of blurring the image in (a) with an averaging filter prior to resizing. The image is slightly more blurred than (b), but aliasing is no longer objectionable. (Original image courtesy of the Signal Compression Laboratory, University of California, Santa Barbara.)
2D Discrete Fourier Transform
Centering the DFT

In MATLAB, use `fftshift` and `ifftshift`
Centering the DFT

Original

Shifted DFT

DFT (look at corners)

Log of shifted DFT
DFT magnitude of geometrically transformed images

- Translated
- Rotated about center

Same magnitude as original (invariant to translation)
DFT phase of geometrically transformed images

Original

Translated

Rotated about center
Contributions of magnitude and phase to image formation

Phase

IDFT: Phase only (zero magnitude)

IDFT: Magnitude only (zero phase)

IDFT: Boy magnitude and rectangle phase

IDFT: Rectangle magnitude and boy phase
Filtering using convolution theorem

Filtering in spatial domain using convolution

Filtering in frequency domain using product without zero-padding wraparound error

Expected result

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Filtering using convolution theorem

Filtering in frequency domain using product with zero-padding

Inverse Fourier transform

Zero padding

Fourier transform

Product

Gaussian lowpass filter in frequency domain

no wraparound error
Filtering using convolution theorem

Filtering in spatial domain using convolution

Filtering in frequency domain using product

Identical results
Filtering in the frequency domain

• Ideal lowpass filter (LPF)
  – Frequency domain
Filtering in the frequency domain

• Ideal lowpass filter (LPF)
  – Spatial domain

\[ H(u,v) \quad h(x,y) \]
Filtering in the frequency domain

• Gaussian lowpass filter (LPF)
Filtering in the frequency domain

- Butterworth lowpass filter (LPF)
Filtering in the frequency domain

- Ideal LPF
- Gaussian LPF
- Butterworth LPF
Example: character recognition

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Gaussian LPF joins broken characters
Highpass filter (HPF)  
Frequency domain

Ideal HPF

Gaussian HPF

Butterworth HPF
Highpass filter (HPF)
Spatial domain

Ideal HPF  Gaussian HPF  Butterworth HPF

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Filtering in the frequency domain

Ideal HPF  Gaussian HPF  Butterworth HPF
Filtering in the frequency domain

1D

Frequency domain

Spatial domain

Lowpass filter

Sharpening filter
Filtering in the frequency domain

- Lowpass filter
- Highpass filter
- Offset highpass filter

2D
Bandreject filters

- Ideal
- Gaussian
- Butterworth
Filtering in the frequency domain

• Sharpening filter
Overview: Image processing in the frequency domain

Image in spatial domain $f(x,y)$ → Fourier transform → Image in frequency domain $F(u,v)$

Frequency domain processing

Image in frequency domain $F(u,v)$ ↓

Image in spatial domain $g(x,y)$ ← Inverse Fourier transform ← Image in frequency domain $G(u,v)$

Jean-Baptiste Joseph Fourier 1768-1830

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Next Lecture

• Image restoration
• Reading
  – Chapter 5: Image Restoration and Reconstruction