Sampling and Aliasing, and The Discrete Fourier Transform

Image Processing
CSE 166
Lecture 6
Announcements

• Assignment 2 is due today, 11:59 PM
• Assignment 3 will be released Apr 22
• Reading
  – Chapter 4: Filtering in the Frequency Domain
Overview: Image processing in the frequency domain

Image in spatial domain $f(x,y)$ \rightarrow Fourier transform \rightarrow Image in frequency domain $F(u,v)$

Frequency domain processing

Image in frequency domain $F(u,v)$ \rightarrow Inverse Fourier transform \rightarrow Image in spatial domain $g(x,y)$

Jean-Baptiste Joseph Fourier 1768-1830
1D impulse function and impulse train

**Impulse function**

- Continuous
  - \( \delta(t) \)
  - \( \delta(t - t_0) \)
  - \( \delta(x) \)
  - \( \delta(x - x_0) \)

**Impulse train**

- Discrete
  - \( s_{\Delta T}(t) \)
  - \( s_{\Delta X}(x) \)
  - \( \cdots, -3\Delta T - 2\Delta T - \Delta T, 0, \Delta T, 2\Delta T, 3\Delta T, \cdots \)
  - \( \cdots, -3\Delta X - 2\Delta X - \Delta X, 0, \Delta X, 2\Delta X, 3\Delta X, \cdots \)
Sampling

![Graph showing sampling process]

**Figure 4.5**
(a) A continuous function. (b) Train of impulses used to model sampling. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of impulses. (The dashed line in (c) is shown for reference. It is not part of the data.)
Sampling

Fourier transform of function

Fourier transforms of sampled function

Over-sampled

Critically-sampled

Under-sampled
The sampling theorem

Fourier transform of function

Fourier transform of sampled function

Critically-sampled
Recovering $F(\mu)$ from $\tilde{F}(\mu)$

Fourier transform of sampled function

Over-sampled

Ideal lowpass filter

Product of above

Recovered
Aliasing

Continuous

Discrete

Under-sampled

Different

Over-sampled

Sampled at same rate

Sampled at same rate

Identical

Alias: a false identity
Aliasing

**Figure 4.11** Illustration of aliasing. The under-sampled function (dots) looks like a sine wave having a frequency much lower than the frequency of the continuous signal. The period of the sine wave is 2 s, so the zero crossings of the horizontal axis occur every second. $\Delta T$ is the separation between samples.
Aliasing

Fourier transform of under-sampled function

Ideal lowpass filter

Product of above

Imperfect recovery

Interference
Next Lecture

• Filtering in the frequency domain
• Reading
  – Chapter 4: Filtering in the Frequency Domain